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Visualization of the Geometry Problems in Primary Math Education - Needs and Challenges -

Tatjana Atanasova – Pachemska[1] , Vesna Gunova [2], Limonka Koceva Lazarova[3], Sanja Pachemska[4],

[1,2,3] Faculty of Computer science, Goce Delcev University, Stip, Macedonia,
[4] Bureau of Educational Development of the Republic of Macedonia

Abstract: The visualization as an approach in the teaching process is not only limited representation of drawings in order to illustrate certain objects or concepts, but it is used in every step of solving mathematical problems, especially in the geometry. It is thought that mathematics is more "abstract world", which examines objects and concepts quite different from physical phenomena that rely on visualization with all its various forms and levels. In this paper we analyzed the perceptions and attitudes about the use of ICT tools for visualization as a "modern" approach for solving geometry problems in primary schools in Macedonia. The obtained results of the research are processed with the software package SPSS19. Observations are discussed and there are given conclusions and recommendations.

Keywords: visualization, geometry, mathematical problem, formalism, ICT tool.

INTRODUCTION

Visualization in mathematics education

In [6], visualization has been described as the creation of a mental image of a given concept. As such, and from the teaching point of view, visualization seems to be a powerful method to utilize for enhancing students' understanding of a variety of concepts in many disciplines such as computer science, chemistry, physics, biology, engineering, applied statistics and mathematics, [10]. Specifically, there are many reasons that substantiate the use of visualization for learning and teaching of mathematics at all levels of schooling, from elementary to university passing through the middle and high school levels. The literature also indicates that the activity of 'seeing' differently is not a self-evident, innate process, but something created and learned ([7], [13]). As cognitive science suggests, we learn to see; we create what we see; visual reasoning or 'seeing to think' is learned, it can also be taught and it is important to teach it ([7], [14]). Thus, teachers who have learned and became skillful in the use of visualization and 'seeing to think' would be able to reinforce mathematical concepts and improve the learning process in the classroom.

Visualization in mathematics is the process of forming images (mentally, using pencil and paper or by using some technology) and effective implementation and use of these images for mathematical discovery and understanding of some mathematical problem. The researchers usually discuss the mathematical visualization in a figurative sense as "seeing the invisible" ([5], [9], [12]). Visual justification in mathematics refers to the understanding and application of mathematical concepts using

[1] tatjana.pacemska@ugd.edu.mk

visually based representations and processes presented in diagrams, computer graphics programs and physical models.

Visualization is not only limited to representation of drawings to illustrate certain objects or concepts, but it is also used in every step of solving any mathematical problem. Many students in the process of solving problems in mathematics don't have the ability for appropriate visual representation and therefore they can not find an appropriate and correct solution for the problem.[1], [2], [3], [4].

The use of visualization tools to support students' exploration of mathematical concepts is not new because they are thought to encourage higher levels of engagement. Popular systems for visualization have brought many benefits to the teaching and learning of mathematics in general, and geometry in particular. Most of the benefits, however, seem to be analytical and suggestive rather than derived from empirical evaluations of these tools, [8].

With computers, we can create not only narrative animations – telling a fixed story by navigating along a chosen path through a parameter space – but also interactive animations, allowing the viewer to make her own explorations. Even at the level of 2D geometric diagrams, modern interactive packages have led to new insights into elementary euclidean geometry. Drawing accurate images of 3D geometric objects by hand requires care and training in perspective. But these are easy to create with computer graphics, and it is trivial even to render a pair of images for stereoscopic viewing with the left and right eye, [11].

METHODOLOGY

Visualizing problems of geometry in primary education

One of the primary goals of this research is determination of the views and teachers' opinion in primary education in the Republic of Macedonia in terms of visualization in mathematics, especially in geometry. It is important to get information whether the teachers encourage the use of this method and how their students can solve the problems in geometry. If it is determined that they use certain computer software for visualizing geometric problems, we research how they do visualization.

Our research is conducted in the period from April to June in 2015 in three primary schools in the Republic of Macedonia in Valandovo, Kavadarci and Stip. The survey was conducted on 21 teachers.

This research is experimental and it monitors the effectiveness of the teaching geometry by means of visualizing in different places and in different conditions. Two groups of students were selected. The first selected group of students was educated by the teacher who encourages visualizing problems of geometry (using ICT) and a group of students with a teacher who declared in the questionnaire that visualizing geometric problems confuses the students, believing that it is an individual desire to resolve the problems.

There are two classes of students in seventh grade who have approximately same average grade in mathematics. Two groups (control and experimental) of 30 students are tested about the process of teaching geometry and solving geometric problems with or without visualization of the tasks, after the conducted survey of teachers. This testing was done in order to see if the visualization in the geometry problems influences on educational success of these students for teaching the subject of geometry.

The goals of the research are:

- determination of the teachers attitudes associated with the use of visualization as a tool which simplifies problems in geometry;
- to determine if the teachers influence the students for using visualization in geometry;
- to determine how teachers implement visualization for geometric problems and whether they use relevant IT tools or not;
- to determine if the teachers who do the teaching geometry by visualizing the problems from geometry have students who have greater success compared with students who did not apply visualization to solve the problem of geometry.

In order to determine whether the success of learning geometry that is achieved by students in the control and experimental group depends on visualizing geometric problems, we set the following hypotheses:

H_0: There is no statistically significant difference between the success in the geometry test that is achieved by students who visualize problems and students who do not visualize geometric problems.

H_1: There is a statistically significant difference between the success in the geometry test that is achieved by students who visualize problems and students who do not visualize geometric problems.

The additional hypothesis is:

H_A: Teachers encourage the use of visualization in solving geometry problems.

RESULTS AND DISCUSSION

From this research in which we use a survey on 21 teachers in three primary schools in the Republic of Macedonia, the following results are obtained.

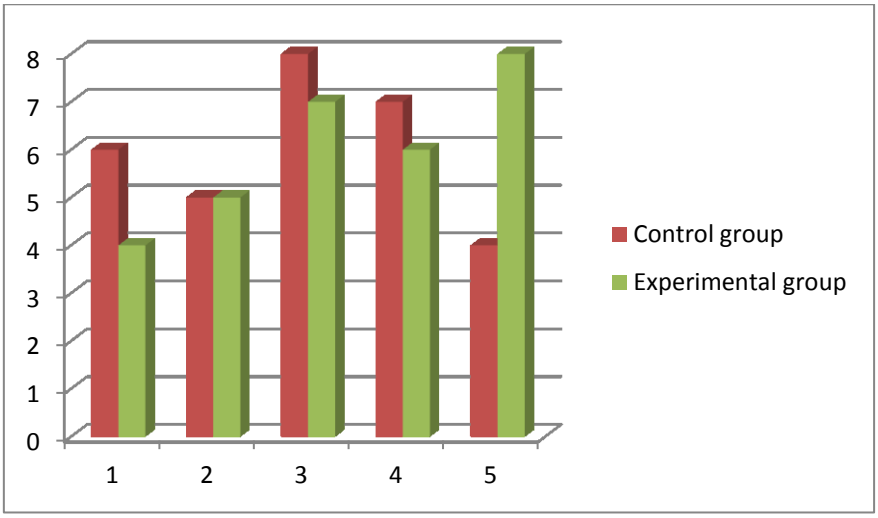
Regarding the attitudes of the teachers about visualization of a problem in the geometry, 19 of 21 expressed a positive attitude. They explained that by the visualization of any problem in the mathematics, especially in the geometry they are getting simplification and easier solution. Two teachers have a negative attitude. They have explained that students are confused when they use visualization of some problem, especially complex problem in the geometry. They explained they prefer to use ready-made formulas and thus without visualization of the problem to get the solution.

Regarding the question, should their students apply visualization in the process of solving problems in geometry and should they encourage the students to do that, 15 teachers responded "yes". They consider that the students' ability for visualization is a positive value. Also, they responded that they motivate their students to do visualization. The visualization is the only way to simplify the problem and to obtain additional directions from the illustration how to solve it. Six teachers from the total surveyed believe that not every student has the "skill" to visualize. According to them the skill for visualization depends on many factors such as gender, student's age, educational success and others.

On the questions of how teachers perform the visualizing of the problems of geometry, 9 teachers answered that they do visualization with the application of ICT in teaching and that computer is a teaching tool in their daily operations. They believe that by using ICT in teaching mathematics for visualization, their students get a clearer picture and further improve and expand their knowledge. They point out that the programming software Geogebra is also good and appropriate for using in other thematic units not only for geometry.

Six teachers who have implemented the visualization in the mathematical education, announced that their students most of the time, make a sketch of the problem with a pen on paper. These teachers occasionally include ICT in the mathematical education and consider that the application of ICT in the mathematical education results with long lasting knowledge.

In addition we are presenting the results from this research which was implemented on the students. From the conducted testing of the two classes with the same number of students and very similar average grade before testing, we received results, which are presented with the following histogram.



This histogram presents the number of students and the grades from the test in geometry are given. We can see the results for the experimental group (where the students visualized the problems from the test) and the results for the control group (geometrical problems were solved only with formulas without visualizing).

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
ControlGroup	30	1	5	2,93	1,337
ExperimentalGroup	30	1	5	3,30	1,393
Valid N (listwise)	30				

Table 1.

We can note that the average grade in geometry test is higher in the experimental group where the students visualized every geometrical problem.

Student's T-test

To test our hypothesis we used Student's t-test for repeated measurements. The testing is repeated in two groups, control and experimental. In the control group the tasks are resolved without visualizing and in the experimental the visualization of the geometry problems is used. Analysis of the results is made with SPSS19.

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference Lower	Upper			
Pair 1	ControlGroup – Experimental Group	-,367)	,490	,089	-,550)	-,184)	-4,097)	29	,000

Table 2

From this analysis, the obtained values show the probability of making wrong conclusion. If the value is smaller than 0,05, we conclude that there is a statistically significant difference. In our case Sig (2-tailed) =0, which means that there is a statistically significant difference between the achieved success of the students and the realization method of mathematical teaching. The null hypothesis is dismissed and the alternative is accepted.

If we calculate the value of square η , with the help of t, because the size of the statistical difference, we get a value of 0,38, therefore we can conclude that *there is a big statistical difference between the success of the students who visualize the geometry problems and the success of the students who solve geometry assignments without visualization.*

CONCLUSION

From this research we can conclude that the teachers have a positive attitude towards visualization in the geometry problems. They are aware of the advantages of the ICT application in

mathematical teaching, especially for visualization in the geometry. But, also some of them are not interested to realize it. They do not insist and encourage their students to use this technique.

The application of ICT in solving complex problems is so important. It is expected from the teachers in mathematics to be leaders in implementing new methods and strategies for learning and solving mathematical problems especially geometrical problems.

From the conducted research we concluded that the application of the visualization and its implementation in the process of solving geometry problems brings significantly positive changes. The teachers say that the visualization, especially the dynamics that the educational software offers, and the independent researching on the internet results with deeper and long lasting knowledge. But, many of the teachers are supporters of the traditional education where the students learn facts and formulas while solving geometry problems instead of using new methods and strategies.

The students are holders of development in our society. They should be guided and directed by their teachers to use the potential of the modern technology in order to achieve permanent and applicable mathematical knowledge.

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