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Population and economic growth theme: Longitudinal data for a sample of Balkan countries

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Abstract

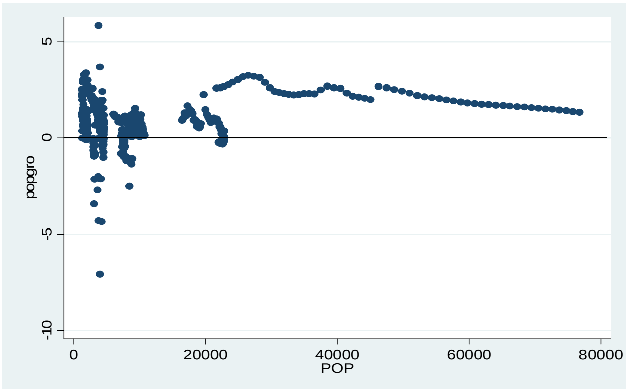
In this paper we use pooled cross-sectional (longitudinal data) in a sample of 10 Balkan countries. The period we cover is from 1950-2009 data are for population and economic growth. In the theoretical part we present optimal intergenerational model of population growth .The optimal population growth depends on capital in the future period and future consumption. Consumption should be greater than zero, and less than total capital of the current generation. In the econometric part OLS regression with dummies the coefficient on Macedonia, is highest significant coefficient meaning, if we control for Macedonia we will on average find more positive association between growth of GDP and population growth. Hausman test was in favor of fixed effects model, but fixed effects and Random effects model showed that there is positive coefficient between GDP growth and population growth. Coefficient in the FE model was statistically significant, which was not case in RE model. From the Fischer's panel unit root test we reject the null hypothesis that panels contain unit root and we accept the alternative that at least one panel is stationary, for the population growth and GDP growth.

Keywords: Population growth, economic growth, Fixed effects model, Random effects model, OLS with dummies model

Introduction

In the beginning of the theoretical section we will start with (Kremer, (1993))[1] evidence that the relationship between population growth and population is almost linear but also statistically significant. In this section we will use our data on population and population growth (See Section data and methodology for explanations)[2].This data cover 10 Balkan countries ,panel data that cover time period for every of the 10 Balkan countries from 1950 to 2009 The level and growth population are presented in the next scatter

Scatter level of population and population growth



This figure shows strongly positive and as we will see statistically significant relationship between population (in thousands) and growth of population.

A regression on a constant and population (in thousands) yields (See Appendix 1)[3]:

$$popgro = 0.58 + 0.0000196\,pop \qquad (1)$$

(0.000) (0.000)

R^2=0.06

Here *popgro* is population growth and *pop* is population in thousands, score is positive and statistically significant at all levels of conventional significance. On the next 2 tables we present the data on GDP and Population growth for the 10 Balkan countries from 2001-2010.

[1] Michael Kremer (1993), "Population Growth and Technological Change: One Million B.C. to 1990," *Quarterly Journal of Economics* 108:3 (August), pp. 681-716.

[2] See Section data and methodology for explanations.

[3] See Appendix 1 Regression on population growth and level of population

Table 1 Population growth in 10 Balkan countries for the period 2001 -2010[4]

Country Name	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Albania	0.18	0.40	0.55	0.58	0.54	0.47	0.41	0.37	0.36	0.36
Bosnia and Herzegovina	1.47	0.73	0.18	-0.04	-0.01	0.02	-0.07	-0.13	-0.17	-0.20
Bulgaria	-1.88	-0.52	-0.59	-0.54	-0.53	-0.53	-0.51	-0.48	-0.50	-0.55
Croatia	0.32	0.00	0.00	-0.02	0.07	-0.05	-0.09	-0.05	-0.11	-0.11
Greece	0.30	0.34	0.33	0.35	0.38	0.40	0.40	0.40	0.41	0.32
Macedonia, FYR	0.35	0.31	0.27	0.26	0.25	0.24	0.24	0.22	0.21	0.18
Romania	-1.40	-1.50	-0.28	-0.26	-0.23	-0.22	-0.19	-0.15	-0.15	-0.18
Serbia	-0.17	-0.05	-0.26	-0.23	-0.30	-0.39	-0.41	-0.43	-0.40	-0.39
Slovenia	0.15	0.10	0.09	0.07	0.18	0.32	0.56	0.16	0.90	0.64
Turkey	1.43	1.39	1.36	1.34	1.34	1.34	1.34	1.32	1.29	1.25

Source: World Bank

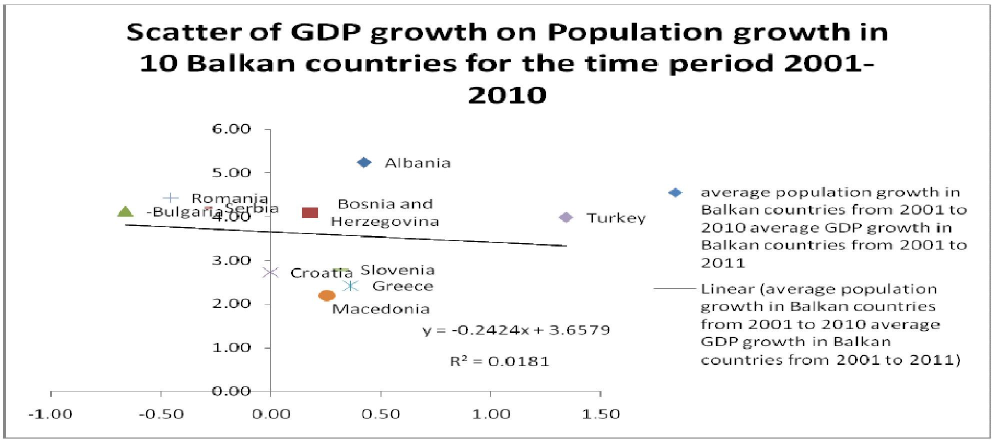
Table 2 GDP growth in 10 Balkan countries for the period 2001-2010

Country Name	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Albania	7.00	2.90	5.70	5.90	5.50	5.00	5.90	7.70	3.30	3.50
Bosnia and Herzegovina	4.40	5.30	4.00	6.10	5.00	6.20	6.84	5.42	-3.10	0.80
Bulgaria	4.15	4.65	5.51	6.75	6.36	6.51	6.45	6.22	-5.52	0.20
Croatia	3.66	4.88	5.37	4.13	4.28	4.94	5.06	2.17	-5.99	-1.19
Greece	4.20	3.44	5.94	4.37	2.28	5.17	4.28	1.02	-2.04	-4.47
Macedonia, FYR	-4.53	0.85	2.82	4.09	4.10	3.95	5.90	5.00	-0.90	0.70
Romania	5.70	5.10	5.20	8.40	4.17	7.90	6.00	9.43	-8.50	0.95
Serbia	5.60	3.90	2.40	8.30	5.60	5.23	6.90	5.52	-3.12	1.76
Slovenia	2.85	3.97	2.84	4.29	4.49	5.81	6.80	3.49	-7.80	1.18
Turkey	-5.70	6.16	5.27	9.36	8.40	6.89	4.67	0.66	-4.83	8.95

Source: World Bank

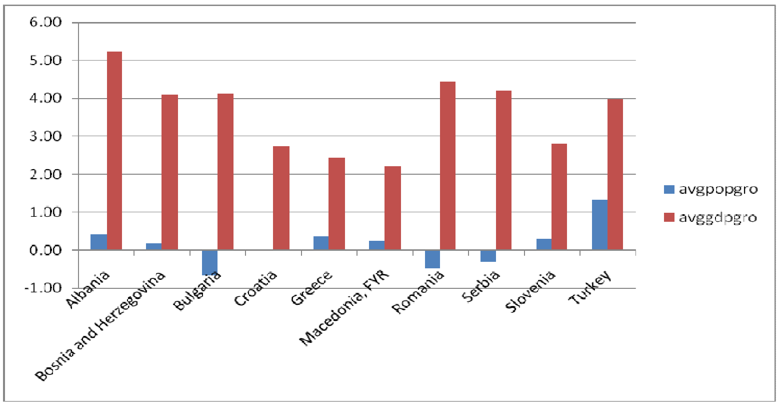
On the next scatter are presented average growth rates of population and GDP , we add a linear trend to the scatter and GDP growth is negatively correlated with the population growth by -0.24 and intercept is 3.65 .This means that if population increases by 1 percentage point GDP growth on average will decline by 0.24 percentage points.

Scatter GDP growth on population growth



[4] These data are gathered from World Bank data base: http://data.worldbank.org/country.

Population growth rate is very slow in the Balkans.Especially in Bulgaria (-0.66), Romania (-0.46), Serbia(-0.30), have negative population growth rate (see chart below).Croatia (0.0) doesn't have population growth, Bosnia and Herzegovina (0.18), Macedonia (0.25), Greece(0.36), Slovenia (0.32), Albania (0.42) and Turkey(1.34).



The demographic structure will be very old in the next decades. This can bring social security problems similar to those of Germany and the other Western European countries.Albania has highest average GDP growth (5.24), followed by Romania(4.43), Serbia(4.21), Bulgaria(4.13), Bosnia and Herzegovina (4.10), Slovenia(2.79), Croatia(2.73), Greece (2.42), Macedonia (2.20). Macedonia has lowest GDP growth from 2001-2010.

Population growth theories

Malthus prediction, made in 1801 that population growth would run up against the fixity of earth's resources and condemn most of the population to poverty and high death rates proved wrong. Kuznets defined growth in 1966 as sustained increase in population attained without any lowering of per capita product, and viewed population growth as positive contributor to economic growth (Birdsall,N.,(1988)[5].

Table 3 Natural increase in population in the World by economies and regions

Birth and death rates of natural increase , by region, 1950-1955 to 1980-85									
	Crude birth rate			Crude death rate			Natural increase		
	1950-55	1960-65	1980-85	1950-55	1960-65	1980-85	1950-55	1960-65	1980-85
Developed countries	22.7	20.3	15.5	10.1	9.0	9.6	1.3	1.1	0.6
Developing countries	44.4	41.9	31.0	24.2	18.3	10.8	2.0	2.4	2.0
Africa	48.3	48.2	45.9	27.1	23.2	16.6	2.1	2.5	2.9
Latin America	42.5	41.0	31.6	15.4	12.2	8.2	2.7	2.9	2.3
East Asia	43.4	39.0	22.5	25.0	17.3	7.7	1.8	2.2	1.5
Other Asia	41.8	40.1	32.8	22.7	18.2	12.3	1.9	2.2	2.1

Source: United Nations, Department of International Economic and Social Affairs, World population prospects as assessed in 1984(printout).

[5] Birdsall, N., (1988), Handbook of development economics ,Volume 1, edited by T.N.Srinivasan

Since 1950's population growth in developing countries has been around 2.0. Most of the Balkan countries belong to this group except Greece that is advanced economy according to IMF and Slovenia (developing country before 2007). In the developed economies since 1950's we have population growth slowdown to 0.6 in the end of 1980's. In the regions Africa has achieved growth in population, Latin America had declined in population growth, and Other than East Asia the other parts of Asia had increased population growth to 2.1 in the end of 1980's. The population growth rate for the developing countries as well for the world, is predicted to decline towards zero rate bringing population stabilization in the twentieth second century[6].Even with population growth rate decline size of population in the developing countries will continue to rise, and world population to reach 10 billion before 2050. For the next few decades the variance of prediction is small, so we cannot be sure about the precision of these demographic predictions. Industrial countries according to some projections will increase their population for 20% by 2050, and developing countries will double their population by 2050. Assaf Razin and Uri Ben-Zion(1993) have outlined intergenerational model of population .Population was included in social utility function and assumption was made that preferences are same for each generation:

$$V = \sum_{t=0}^{\infty} \beta^t U(c_t, \lambda_t) \qquad (2)$$

Here β is the subjective factor by which current generation discounts utility of the next generation. The inclusion of population growth in the social utility function has also an empirical implication for the measurement of welfare improvement. That is, growth of per capita income, by itself, is an inappropriate measure of welfare improvement, and as a measure it is biased against countries with a high rate of population growth. The decision problem for current generation can be written as :

$$V(k_0) = \max\left\{\sum_{t=0}^{\infty} \beta^t U\left(c_t, \lambda_t\right)\right\} \qquad (3)$$

$$0 \le c_t \le k_t$$

$$0 \le \lambda \le \overline{\lambda}$$

[6] Based on the population projections by World Bank

K_t is the capital for the current generation; λ_t is the current level of population growth $\bar{\lambda}$ is the maximum feasible level of population growth. Marginal utilities are positive and diminishing. c_t is per capita life time consumption. Following decision is presented partially derived:

$$\frac{\partial U}{\partial \lambda}(c_t, \lambda_t) = \frac{\beta}{\lambda_t} k_{t+1} \frac{\partial U}{\partial c}(c_{t+1}, \lambda_{t+1}) \quad (4)$$

$$\frac{\partial U}{\partial \lambda}(c_t, \lambda_t) = \frac{\beta}{\lambda_t} \frac{\partial f}{\partial k}(k_t - c_t) \frac{\partial U}{\partial c}(c_{t+1}, \lambda_{t+1}) \quad (5)$$

Equation (4) may be interpreted as describing the optimum decision with respect to the level of population growth λ_t On the one hand an extra unit of λ_t will increase welfare by the marginal utility of population growth, the left-hand side of (4). In the second equation the level of capital is decreased by the consumption of the current generation. And this equation (5) describes the optimal level of consumption.

According to Ramsey (1928)[7], optimal rate of consumption is:

$$u(c) = \frac{dU(c)}{dc} \quad (6)$$

In the equilibrium there will be no saving and

$$\frac{dc}{dt} = \frac{dk}{dt} = 0 \quad (7)$$

Marginal productivity of capital is :

$$\frac{\partial f}{\partial k} = \rho \quad (8)$$ [8]

If we take into account intergenerational differences in tastes we get:

$$U(c_0, \lambda_0) = a \log c_0 + v(\lambda_0) \quad (9)$$

$$U(c_t, \lambda_t) = a \log c_t + v(\lambda_t, \theta), t \geq 1 \quad (10)$$

Here □ is parameter in the function v which distinguishes the utility of future generations, derived from population increase, from that of the parents generation .If we include uncertainty in the population growth we get :

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[7] Ramsey,F.,P.(1928), *A Mathematical theory of saving*, The Economic journal Vol.38 No.152

[8] ρ is the rate of discounting if $\frac{\partial f}{\partial k} > \rho$ there will be saving ,or investment $\frac{\partial f}{\partial k} < \rho$

$$V(k_0) = E\left\{\sum_{t=0}^{\infty} \beta^t U(c_t, \lambda_t))\right\} \tag{11}$$

$$0 \le c_t \le k_t$$

$$0 \le h_t \le \overline{h}$$

Here E is the expected value of the population growth, expectation operator. Consumption should be greater than zero, and less than total capital of the current generation, and h_t is the variable by which population change is controlled.

Empirical part

Econometric Methodology

Data in this paper are gathered from Penn world Table[9]. Data cover period from 1950 to 2009 for 10 Balkan countries: **Albania, Bosnia and Herzegovina, Bulgaria, Croatia, Greece, Macedonia, Romania, Serbia, Slovenia, Turkey**. These are 10 panels 60 observations per panel. But the data set has gaps on average we have 59,6 observations per group, so in 10 panels we have around 596 observations. Mostly data are missing for the GDPPPP (GDP in PPP terms) for the period 1950 to 1969 this is due to lack of data collection by the statistical bureaus in this countries for this period.

These data are pooled cross-section time series or panel data. Pooled data are characterized by having repeated observations (most frequently years) on fixed units (most frequently states and nations). This means that pooled arrays of data are one that combines cross-sectional data on N spatial units and T time periods to produce a data set of $N \times T$ observations (Podestà,2002). However, when the cross-section units are more numerous than temporal units (N>T), the pool is often conceptualized as a "cross-sectional dominant". conversely, when the temporal units are more numerous than spatial units (T>N), the pool is called "temporal dominant" (Stimson 1985). The generic pooled linear regression model estimable by Ordinary Least Squares (OLS) procedure is given by the following equation:

$$y_{it} = \beta_1 + \sum_{k=2}^{k} \beta_k x_{kit} + e_{it} \tag{12}$$

$$\Delta y_i = \delta_0 + \beta_1 \Delta x_i + \Delta u_i \tag{13}$$

where "Δ" denotes the change from t =1 to t = 2. The unobserved effect, a_i, does not appear in (2): it has been "differenced away." Also, the intercept in (2) is actually

[9] http://pwt.econ.upenn.edu/php_site/pwt70/pwt70_form.php **Alan Heston, Robert Summers and Bettina Aten, Penn World Table Version 7.0, Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania, May 2011.**

the change in the intercept from t =1 to t =2.Equation (2) is simple first differenced pooled cross section regression where each variable is differenced over time. After we apply OLS estimation we will run fixed effects and random effects model

Static two way fixed effect model:

$$y_{it} = \alpha_i + \delta_i t + \rho y_{t-1} + \theta_t + e_{it} \qquad (14)$$

$$i = 1,...N \quad t = 1,...T \qquad (15)$$

1. α_i unit-specific characteristics
2. γ_i unit-specific deterministic trend parameters
3. μ_t time-specific effects (common to all units)
4. β is common to all units

Next random effects model also is going to be applied. If you have reason to believe that differences across entities have some influence on your dependent variable then you should use random effects.

The random effects model is :

$$Y_{it} = \beta X_{it} + \alpha + u_{it} + \varepsilon_{it} \qquad (16)$$

u_{it} is between entity error, ε_{it} is within entity error.

Unobserved model becomes random effects model when we assume that unobserved effect α is uncorrelated with each explanatory variable:

$$\text{cov}(x_{itj}, \alpha_i) = 0, t = 1,2......T; j = 1,2,..., K \qquad (17)$$

If we define composition error term $v_{it} = \alpha_i + u_{it}$:

$$y_{it} = \beta_0 + \beta_1 x_{it1} + ... + \beta_k x_{itk} + v_{it} \qquad (18)$$

Im, Pesaran and Shin (JE 2003) propose a test based on the average of a augmented Dickey-Fuller tests computed for each panel unit in the model

$$y_{it} = \alpha_i + \delta_i t + \rho y_{it-1} + \theta_t + e_{it} \qquad (19)$$

where e_{it} can be:

- Serially correlated
- and heteroscedastic
- but cross-sectional independent apart from the presence of the common time effects θ_t .

The estimating equation is :

$$\Delta y_{it} = \phi_i y_{it-1} + \sum_{k=1}^{K_I} \gamma ki \Delta y_{it-k} + \varepsilon_{it} \qquad (20)$$

The null hypothesis of a unit root is tested using $t_{bar} = \frac{1}{N}\sum_{i=1}^{N} t\phi i$

$H_0 : \phi = 0$

against the heterogeneous alternative:

$$H_1 : \begin{cases} \phi < 0 \; for\, i = 1,....N_1 \\ \phi = 0 \; for\, i = N_1 + 1,....N \end{cases} \tag{21}$$

In the panel unit root test in the general model, let us first look at the test $H_0 = \rho = 1$

H_0: unit root Different H_1 specifications have been proposed for the model:

$$y_{it} = \alpha_i + \delta_i t + \rho y_{it-1} + \delta_i \theta_t + \varepsilon_{it} \tag{22}$$

$$H_1 : \begin{cases} \rho < 1 \quad for \quad all \quad i \\ \rho = 1 \quad for \quad i = N_1 + 1,....N \end{cases}$$

Data

To estimate the following model we define the following set of variables:

Table 1 Variable definitions

Variable	Definition
lgdpgro	Logarithm of growth of GDP per capita PPP converted at 2005 constant prices
lpopgro	Log of growth rate of population in thousands

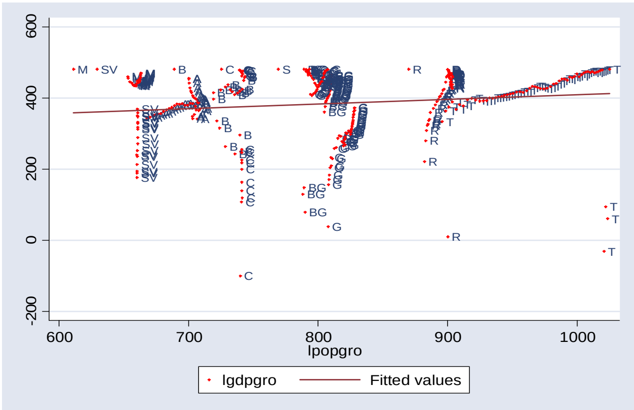
Descriptive statistics of the model

In the descriptive statistics we report the usual number of observations per variable, means, standard deviations, and minimums and maximums. The descriptive statistics of our model for ten countries is given below in a Table 2.

Table 2 Descriptive statistics of the model

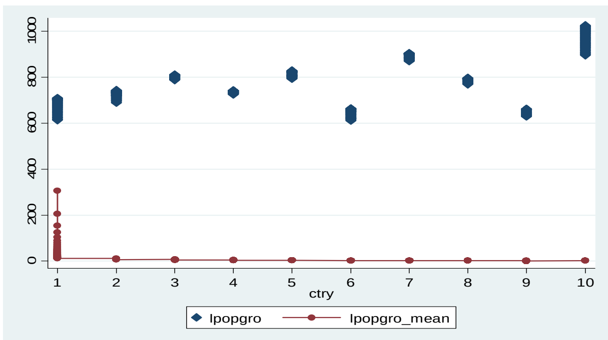
Variable	**Obs.**	**Mean**	**Std.deviation**	**Min**	**Max**
lgdpgro	342	384.5786	98.82886	-100	481.413
lpopgro	596	770.1818	101.867	611.0394	1024.904

For the table of the descriptive statistics of the model we can see that the mean of log of population growth is 770.1818 (thousands), minimum is 611.0394(thousands) while the maximum of this variable is 1024.904(1 million and 24 thousands and 904) . Visually from the next graph we can see that lgdpgro and lpopgro are positively correlated. On this plot we use acronyms for the 10 countries (**Albania-A, Bosnia and Herzegovina-B, Bulgaria-BG, Croatia-C, Greece-G, Macedonia-M, Romania-R, Serbia-S, Slovenia-SV, Turkey-T).**



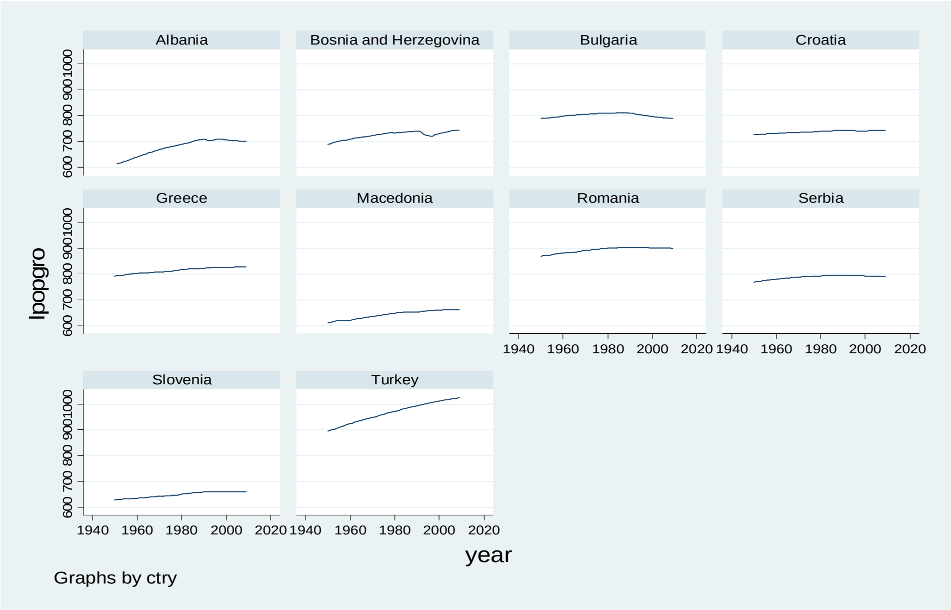
From the graph we can see that substantial part of the observations is below the trend in logarithm of the GDP per capita growth and Turkey has highest population growth from the sample countries while Macedonia some of the lowest, and Croatia and Turkey have experienced negative GDP growth rates. When we try to investigate heterogeneity across countries or entities we do so by creating scatter two way for population growth and country. The resulting scatter from our data I given on the next page. There countries are numbered: **1.Albania 2. Bosnia and Herzegovina, 3.Bulgaria,4. Croatia, 5.Greece,6. Macedonia,7. Romania,8.Serbia, 9.Slovenia, 10. Turkey.**

Scatter: Fixed effects: Heterogeneity across countries (or entities)

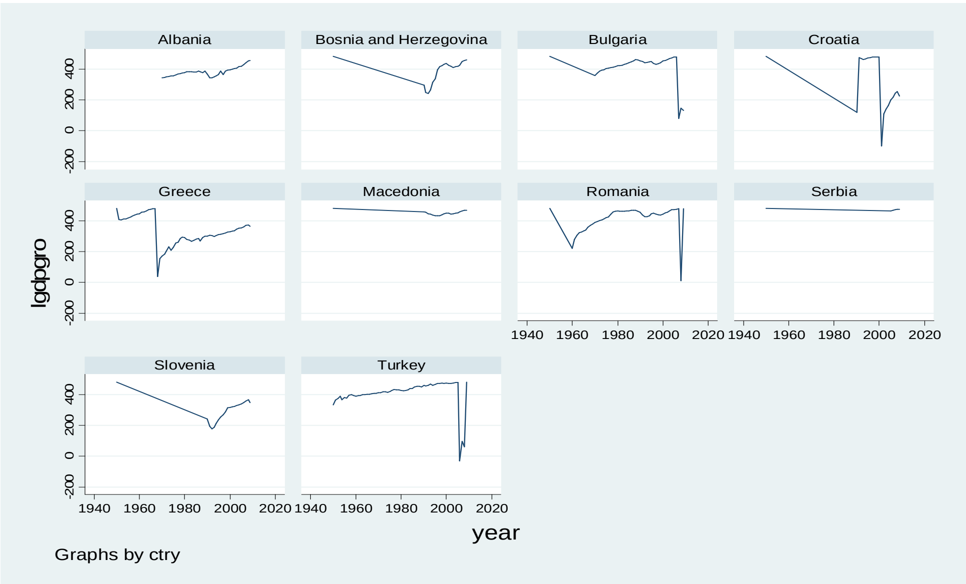


On the scatter is presented logarithm of population growth mean for the 10 countries. Turkey has highest population growth, while Macedonia lowest in the region, together with Slovenia that has little higher growth of population. Log of population growth across Balkan countries si given in the following table of graphs 3

Table of graphs 3



We can create a Table of graphs even for log of GDP per capita growth **Table of graphs 4**



From the scatter we can see that countries like Croatia, Bulgaria, Turkey, Romania have suffered from the economic and financial crisis circa 2007-2008, with a sharp decline in the log of growth of GDP variable.

Least squares dummy variable model (LSDV)

There are several strategies for estimating fixed effect models. The least squares dummy variable model (LSDV) uses dummy variables, whereas the within effect does not. These strategies produce the identical slopes of non-dummy independent variables. The between effect model also does not use dummies, but produces different parameter estimates. There are pros and cons of these strategies .These are presented in the following table

Table 5 Pros and cons of different ways of estimating fixed effects model [10]

	LSDV1	Within effect	Between effect
Functional form	$y_i = i\alpha_i + X_i\beta + \varepsilon_i$	$y_{it} - \bar{y}_{in} = x_{it} - \bar{x}_{in} + \varepsilon_{it} - \bar{\varepsilon}_{in}$	$\bar{y}_{in} = \alpha + \bar{x}_{in} + \varepsilon_i$
Dummy	Yes	No	No
Dummy coefficient	Presented	Need to be computed	N/A
Transformation	No	Deviation from the group means	Group means
Intercept	Yes	No	No
R^2	Correct	Incorrect	
SSE	Correct	Correct	
MSE	Correct	Smaller	
Standard error of β	Correct	Incorrect(smaller)	
DF_{error}	nT-n-k	nT-n-k(Larger)	n-K
Observations	nT	nT	n

Testing for group effects

The null hypothesis is that all dummy parameters except one are zero:

$$H_0 : \mu_1 = ... = \mu_{n-1} = 0 \qquad (23)$$

This hypothesis is tested by the F test (Greene ,2008)[11], which is based on loss of goodness-of-fit. The robust model in the following formula is LSDV and the efficient model is the pooled regression.

$$F(n-1, nT-n-K) = \frac{\left(R^2_{LSDV} - R^2_{Pooled}\right)/(n-1)}{\left(1-R^2_{LSDV}\right)/(nT-n-K)} \qquad (24)$$

[10] Source: Indiana University Stath/Math center

[11] Greene,H.W.,(2008), Econometric Analysis, Prentice Hall

Here T=total number of temporal observations. n=the number of groups, and k=number of regressors in the model. If we find significant improvements in the R^2, then we have statistically significant group effects.

In Greene (2008) this model in matrix notation is presented as:

$$y = [x \quad d_1 \quad d_2 .... d_n] \begin{bmatrix} b \\ a \end{bmatrix} + \varepsilon \quad (25)$$

With assembling all nT rows gives:

$$y = X\beta + D\alpha + \varepsilon \quad (26)$$

Table 6 OLS regression and OLS with dummies (Appendix 2) [12]

Dependent variable: lgdpgro	Logarithm of growth of GDP per capita PPP	Ordinary least squares	Ordinary least squares with dummies
variables		OLS	OLS_dum
lpopgro	Log of growth rate of population	0.13*	0.06
_Icountry_2	Bosnia and Herzegovina		4.81
_Icountry_3	Bulgaria		23.99
_Icountry_4	Croatia		-61.16*
_Icountry_5	Greece		-55.76
_Icountry_6	Macedonia		71.53**
_Icountry_7	Romania		22.48
_Icountry_8	Serbia		86.1
_Icountry_9	Slovenia		-87.8**
_Icountry_10	Turkey		10.79
_cons	Constant	280.31***	341.85
N		339	339
F-statistics (1, 337)			8.40***

legend: * p<0.05; ** p<0.01; *** p<0.001

This OLS model shows that on average in these 10 Balkan countries if the population increases by 1% GDP in these 10 countries will rise by 0.13 percent. This coefficient is signifi-

[12] See Appendix 2

cant at 1% level of significance. Dummy variables take values from [0,1],zero if the country is not included in the regression and 1 if the country is in the regression. Dummies for Croatia, Macedonia, and Slovenia are significant at 1%, 5%, and 10% levels of significance. So for instance coefficient on Macedonia is highest significant coefficient meaning if we control for Macedonia we will on average find more positive association between growth of GDP and population growth. If we include Croatia and Slovenia in the regression growth of population would have been growth detrimental. If Serbia was in the regression we would have on average found more positive association between growth of GDP and population growth, but typically if we control for Serbia in the regression t-statistics will report 0.10 lower. F-statistics is significant at all levels of conventional significance; this means that we can reject H_0: jointly insignificant dummy variables in favor of the alternative jointly significant dummy variables. By adding the dummy for each country we are estimating the pure effect of lpopgro (by controlling for the unobserved heterogeneity)

Fixed effects model [13]

"**. . .** The fixed-effects model controls for all time-invariant differences between the individuals, so the estimated coefficients of the fixed-effects models cannot be biased because of omitted time-invariant characteristics…[like culture,religion, gender, race, etc] "

To see if time fixed effects are needed when running fixed effect model we will use a joint test to see if the dummies for all years are equal to zero.

The linear regression model with fixed effects is

$$y_{it} = \boldsymbol{\beta}'\mathbf{x}_{it} + \alpha_i + \delta_t + \varepsilon_{it},\ t = 1,\ldots,T(i),\ i = 1,\ldots,N, \quad (27)$$

$$E[\varepsilon_{it}|\mathbf{x}_{i1},\mathbf{x}_{i2},\ldots,\mathbf{x}_{iT(i)}] = 0,$$

$$\text{Var}[\varepsilon_{it}|\mathbf{x}_{i1},\mathbf{x}_{i2},\ldots,\mathbf{x}_{iT(i)}] = \sigma^2.$$

We have assumed the strictly exogenous regressors case in the conditional moments, [see Woolridge (1995)]. We have not assumed equal sized groups in the panel. The vector $\boldsymbol{\beta}$ is a

[13]Greene, W.(2001), **Estimating Econometric Models with Fixed Effects ,** *Department of Economics, Stern School of Business, New York University,*

set of parameters of primary interest, α_i is the group specific heterogeneity. We have included time specific effects but, they are only tangential in what follows. Since the number of periods is usually fairly small, these can usually be accommodated simply by adding a set of time specific dummy variables to the model. Our interest here is in the case in which N is too large to do likewise for the group effects. For example in analyzing census based data sets, N might number in the tens of thousands. The analysis of two way models, both fixed and random effects, has been well worked out in the linear case [See, e.g., Baltagi (1995) and Baltagi, et al. (2005).]. A full extension to the nonlinear models considered in this paper remains for further research The parameters of the linear model with fixed individual effects can be estimated by the 'least squares dummy variable' (LSDV) or 'within groups' estimator, which we denote $\mathbf{b}_{LSDV}$. This is computed by least squares regression of $y_{it}^* = (y_{it} - \bar{y}_{i.})$ on the same transformation of $\mathbf{x}_{it}$ where the averages are group specific means. The individual specific dummy variable coefficients can be estimated using group specific averages of residuals. [See, e.g., Greene (2000, Chapter 14).] The slope parameters can also be estimated using simple first differences. Under the assumptions, $\mathbf{b}_{LSDV}$ is a consistent estimator of $\boldsymbol{\beta}$. However, the individual effects, α_i, are each estimated with the $T(i)$ group specific observations. Since $T(i)$ might be small, and is, moreover, fixed, the estimator, $a_{i,LSDV}$, is inconsistent. But, the inconsistency of $a_{i,LSDV}$, is not transmitted to $\mathbf{b}_{LSDV}$ because $\bar{y}_{i.}$ is a sufficient statistic. The LSDV estimator $\mathbf{b}_{LSDV}$ is not a function of $a_{i,LSDV}$. There are a few nonlinear models in which a like result appears.

We will define a nonlinear model by the density for an observed random variable, y_{it},

$$f(y_{it} \mid \mathbf{x}_{i1},\mathbf{x}_{i2},\ldots,\mathbf{x}_{iT(i)}) = g(y_{it}, \boldsymbol{\beta}'\mathbf{x}_{it} + \alpha_i, \boldsymbol{\theta}) \tag{28}$$

where $\boldsymbol{\theta}$ is a vector of ancillary parameters such as a scale parameter, an overdispersion parameter in the Poisson model or the threshold parameters in an ordered probit model. We have narrowed our focus to linear index function models. For the present, we also rule out dynamic effects; $y_{i,t-1}$ does not appear on the right hand side of the equation. [See, e.g., Arellano and Bond (1991), Arellano and Bover (1995), Ahn and Schmidt (1995), Orme (1999), Heckman and MaCurdy (1980)]. However, it does appear that extension of the fixed effects model to dynamic models may well be practical. This, and multiple equation models, such as VAR's are left for later extensions. [See Holtz-Eakin (1988) and Holtz-Eakin, Newey and Rosen (1988, 1989).] Lastly, note that only the current data appear directly in the density for the current y_{it}. We will also be limiting attention to parametric approaches to modeling. The density is assumed to be fully defined.

Many of the models we have studied involve an ancillary parameter vector, $\boldsymbol{\theta}$. No generality is gained by treating $\boldsymbol{\theta}$ separately from $\boldsymbol{\beta}$, so at this point, we will simply group them in the single parameter vector $\boldsymbol{\gamma} = [\boldsymbol{\beta}',\boldsymbol{\theta}']'$. Denote the gradient of the log likelihood by

$$\mathbf{g}_\gamma = \frac{\partial \log L}{\partial \boldsymbol{\gamma}} = \sum_{i=1}^{N} \sum_{t=1}^{T(i)} \frac{\partial \log g(y_{it}, \boldsymbol{\gamma}, \mathrm{x}_{it}, \alpha_i)}{\partial \boldsymbol{\gamma}} \text{ (a } K_\gamma \times 1 \text{ vector)} \quad (29)$$

$$g_{\alpha i} = \frac{\partial \log L}{\partial \alpha_i} = \sum_{t=1}^{T(i)} \frac{\partial \log g(y_{it}, \boldsymbol{\gamma}, \mathbf{x}_{it}, \alpha_i)}{\partial \alpha_i} \text{ (a scalar)} \quad (30)$$

$$\mathbf{g}_\alpha = [g_{\alpha 1}, \ldots, g_{\alpha N}]' \text{ (an } N\times 1 \text{ vector)} \quad (31)$$

$$\mathbf{g} = [\mathbf{g}_\gamma', \mathbf{g}_\alpha']' \text{ (a } (K_\gamma+N)\times 1 \text{ vector).} \quad (32)$$

The full $(K_\gamma+N)\times (K_\gamma+N)$ Hessian is

$$\mathbf{H} = \begin{bmatrix} \mathrm{H}_{\gamma\gamma} & \mathrm{h}_{\gamma 1} & \mathrm{h}_{\gamma 2} & \cdots & \mathrm{h}_{\gamma N} \\ \mathrm{h}_{\gamma 1}' & h_{11} & 0 & \cdots & 0 \\ \mathrm{h}_{\gamma 2}' & 0 & h_{22} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ \mathrm{h}_{\gamma N}' & 0 & 0 & 0 & h_{NN} \end{bmatrix}$$

Estimating the Fixed Effects Model

We could just include dummy variables for all but one of the units. This "sweeps out the unit effects" because when you mean deviate variables, you no longer need to include an intercept term. So the model regresses $y_{i,t}$ – mean(y_i) on $x_{i,t}$ – mean(x_i). This is often called this "with-in" estimator because it looks at how changes in the explanatory variables cause y to vary around a mean within the unit.

Random Effects models

Instead of thinking of each unit as having its own systematic baseline, we think of each in-tercept as the result of a random deviation from some mean intercept. If we have a large N (panel data), we will be able to do this, and random effects will be more efficient than fixed effects. It has N more degrees of freedom, and it also uses information from the "between" estimator (which averages observations over a unit and regresses average y on average x to look at differences across units). If we have a big T (TS-CS data), then the difference between fixed effects and random effects, goes away.

$$y_{i,t} = \mu + \alpha_i + x_{i,t}\beta + e_{i,t} \tag{33}$$

Table 7 Distinguishing between random effects and fixed effects model[14]

Random vs. Fixed	**Definition**
Variables	***Random variable***: (1) is assumed to be measured with measurement error. The scores are a function of a true score and random error; (2) the values come from and are intended to generalize to a much larger population of possible values with a certain probability distribution (e.g., normal distribution); (3) the number of values in the study is small relative to the values of the variable as it appears in the population it is drawn from. ***Fixed variable***: (1) assumed to be measured without measurement error; (2) desired generalization to population or other studies is to the same values; (3) the variable used in the study contains all or most of the variable's values in the population. It is important to distinguish between a variable that is *varying* and a variable that is *random*. A fixed variable can have different values, it is not necessarily invariant (equal) across groups.
Effects	***Random effect***: (1) different statistical model of regression or ANOVA model which assumes that an independent variable is random; (2) generally used if the levels of the independent variable are thought to be a small subset of the possible values which one wishes to generalize to; (3) will probably produce larger standard errors (less powerful). ***Fixed effect***: (1) statistical model typically used in regression and ANOVA assuming independent variable is fixed; (2) generalization of the results apply to similar values of independent variable in the population or in other studies; (3) will probably produce smaller standard errors (more powerful).
Coefficients	***Random coefficient:*** term applies only to MLR analyses in which intercepts, slopes, and variances can be assumed to be random. MLR analyses most typically assume random coefficients. One can conceptualize the coefficients obtained from the level-1 regressions as a type of random variable which comes from and generalizes to a distribution of possible values. Groups are conceived of as a subset of the possible groups. ***Fixed coefficient:*** a coefficient can be fixed to be non-varying (invariant) across groups by setting its between group variance to zero. Random coefficients must be variable across groups. Conceptually, fixed coefficients may be invariant *or* varying across groups.

Estimations of random and fixed effects model

In the next Table we will present the results from the fixed and random effect regressions. We will perform a Hausman test. Here we mention that when we do this panel models and

[14] Newsom USP 656 Multilevel Regression Winter 2006

regressions on our data independent variables are collinear with the panel variable ctry, so we use second panel variable year because we cannot run the regressions otherwise.

Table 8 Fixed effects model and random effects model (See Appendix 3) [15]

Dependent variable: lgdpgro	Logarithm of growth of GDP per capita PPP	Fixed Effects model	Random Effects model
variables		FE	RE
lpopgro	Log of growth rate of population	0.76	0.28
_Iyear_1951	Dummy 1951	-40.99	-56.28
_Iyear_1952	Dummy 1952	-37.999	-52.399
_Iyear_1953	Dummy 1953	-29.76	-43.268
_Iyear_1954	Dummy 1954	-41.07	-53.69
_Iyear_1955	Dummy 1955	-33.03	-44.74
_Iyear_1956	Dummy 1956	-34.37	-45.16
_Iyear_1957	Dummy 1957	-22.94	-32.79
_Iyear_1958	Dummy 1958	-19.70	-28.55
_Iyear_1959	Dummy 1959	-20.83	-28.67
_Iyear_1960	Dummy 1960	-109.62	-112.96
_Iyear_1961	Dummy 1961	-87.74	-90.35
_Iyear_1962	Dummy 1962	-77.88	-79.88
_Iyear_1963	Dummy 1963	-68.69	-70.14
………	………	………	………
_Iyear_2007	Dummy 2007	-149.48174***	-130.11**
_Iyear_2008	Dummy 2008	-188.25289***	-168.84***
_Iyear_2009	Dummy 2009	-106.23162*	-86.79*
_cons	Constant	-132.74	256.91
N		339	339

legend: * p<0.05; ** p<0.01; *** p<0.001

In the time fixed effects model lpopgro is statistically significant t=1,75 at 10% level of significance, the coefficient is positive 0.76 , meaning that 1% increase in growth of population will induce GDP growth of 0.76%. This variable in RE model has not got significant coefficient. We set years as number of dummies here. We set null hypothesis here that all dummies are equal to zero and we test with F statistics. Probability exceeding F statistics is 0,8507[16]

[15] See Appendix 3 Panel estimation techniques

[16] See Appendix 3 testparm

this means that we cannot reject the null that all years coefficients are zero, therefore no time fixed effects are needed. Hausman test is in favor of Fixed effects model i.e. difference in coefficients is not systematic. Probability >chi2=1.000[17]. Coefficients for the years 2007.2008 and 2009 are highly significant but more negative than other years this is due to financial crisis if we controlled only for these three years on average we will get less positive association between GDP growth and population growth.

Panel unit root tests **(See Appendix 4)**

"xtunitroot performs a variety of tests for unit roots (or stationarity) in panel datasets. The Levin-Lin-Chu (2002), Harris-Tzavalis (1999), Breitung (2000; Breitung and Das 2005), Im-Pesaran-Shin (2003), and Fisher-type (Choi 2001) tests have as the null hypothesis that all the panels contain a unit root. The Hadri (2000) Lagrange multiplier (LM) test has as the null hypothesis that all the panels are (trend) stationary. The top of the output for each test makes explicit the null and alternative hypotheses. Options allow you to include panel-specific means (fixed effects) and time trends in the model of the data-generating process"[18]

xtfisher combines the p-values from N independent unit root tests, as developed by Maddala and Wu (1999). Based on the p-values of individual unit root tests, Fisher's test assumes that all series are non-stationary under the null hypothesis against the alternative that at least one series in the panel is stationary. Unlike the Im-Pesaran-Shin (1997) test (ipshin or xtunitroot ips), Fisher's test does not require a balanced panel. This test is based on augmented Dickey-Fuller tests.

Table 9 Panel Unit root tests Variable gdpgro (Growth of GDP)
Ho: All panels contain unit roots
Ha: At least one panel is stationary

Type of statistic	statistic	p-value	Decision
Inverse chi-squared(20) P	49.1548	0.0003	Sufficient evidence to accept H_A
Inverse normal Z	-3.8714	0.0001	Sufficient evidence to accept H_A
Inverse logit t(49) L*	-4.0690	0.0001	Sufficient evidence to accept H_A
Modified inv. chi-squared Pm	4.6098	0.0000	Sufficient evidence to accept H_A

[17] See Appendix 3 Hausman test
[18] Source Stata manual

So we reject the null hypothesis that panels contain unit root and we accept the alternative that at least one panel is stationary.

Table 10 Panel Unit root tests Variable popgro (population growth)
Ho: All panels contain unit roots
Ha: At least one panel is stationary

Type of statistic	statistic	p-value	Decision
Inverse chi-squared(20) P	61.3497	0.0000	Sufficient evidence to accept H_A
Inverse normal Z	-4.5153	0.0000	Sufficient evidence to accept H_A
Inverse logit t(54) L*	-5.0274	0.0000	Sufficient evidence to accept H_A
Modified inv. chi-squared Pm	6.5380	0.0000	Sufficient evidence to accept H_A

So here also we reject the null hypothesis that panels contain unit root and we accept the alternative that at least one panel is stationary. In conclusion population growth and GDP growth are stationary.

Conclusion

This paper confirmed that for the Balkan countries also applies the rule of linear relationship between population growth and population, but also that demographic structure in the Balkan countries will be very old in the next decades. Optimal population growth depends on capital in the future period and future consumption. Turkey has highest population growth, while Macedonia lowest in the region, together with Slovenia that has little higher growth of population. In the OLS regression with dummies the coefficient on Macedonia, is highest significant coefficient meaning, if we control for Macedonia we will on average find more positive association between growth of GDP and population growth. Hausman test was in favor of FE model, but FE and RE model showed that there is positive coefficient between GDP growth and population growth. Coefficient in the FE model was statistically significant, which was not case in RE model. From the Fischer's panel unit root test we reject the null hypothesis that panels contain unit root and we accept the alternative that at least one panel is stationary, for the population growth and GDP growth.

Appendix 1 Regression on population growth and level of population

```
. regress popgro   pop

      Source |       SS       df       MS              Number of obs =     590
-------------+------------------------------           F(  1,   588) =   39.93
       Model |  46.4512362     1  46.4512362           Prob > F      =  0.0000
    Residual |  684.078853   588  1.16339941           R-squared     =  0.0636
-------------+------------------------------           Adj R-squared =  0.0620
       Total |  730.530089   589  1.24028878           Root MSE      =  1.0786

------------------------------------------------------------------------------
      popgro |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         pop |   .0000196   3.11e-06     6.32   0.000     .0000135    .0000257
       _cons |    .575368   .0554657    10.37   0.000      .466433    .6843029
------------------------------------------------------------------------------
```

Appendix 2 OLS and OLS_dummies regression

```
--------------------------------------------
    Variable |      ols          ols_dum
-------------+------------------------------
     lpopgro |  .12929031*      .05814148
 _Icountry_2 |                  4.8024968
 _Icountry_3 |                  23.983916
 _Icountry_4 |                 -61.154368*
 _Icountry_5 |                 -55.759953
 _Icountry_6 |                  71.522809**
 _Icountry_7 |                  22.472556
 _Icountry_8 |                  86.099647
 _Icountry_9 |                 -87.803317**
_Icountry_10 |                  10.780687
       _cons |  280.31333***    341.84296
-------------+------------------------------
           N |        339             339
--------------------------------------------
      legend: * p<0.05; ** p<0.01; *** p<0.001
```

. xi: regress lgdpgro lpopgro i.country

i.country   _Icountry_1-10   (_Icountry_1 for coun~y==Albania omitted)

```
      Source |       SS       df       MS              Number of obs =     339
-------------+------------------------------           F( 10,   328) =    8.40
       Model |   650078.81    10   65007.881           Prob > F      =  0.0000
    Residual |  2537279.52   328   7735.6083           R-squared     =  0.2040
-------------+------------------------------           Adj R-squared =  0.1797
       Total |  3187358.33   338  9430.05423           Root MSE      =  87.952

------------------------------------------------------------------------------
     lgdpgro |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     lpopgro |   .0581415   .2607112     0.22   0.824    -.4547355    .5710185
```

_Icountry_2	4.802497	25.39018	0.19	0.850	-45.14565	54.75064
_Icountry_3	23.98392	33.98436	0.71	0.481	-42.87089	90.83872
_Icountry_4	-61.15437	26.33497	-2.32	0.021	-112.9611	-9.347613
_Icountry_5	-55.75995	35.73427	-1.56	0.120	-126.0572	14.53731
_Icountry_6	71.52281	25.75835	2.78	0.006	20.85039	122.1952
_Icountry_7	22.47256	55.59951	0.40	0.686	-86.90407	131.8492
_Icountry_8	86.09965	45.34624	1.90	0.058	-3.10652	175.3058
_Icountry_9	-87.80332	26.78825	-3.28	0.001	-140.5018	-35.10485
_Icountry_10	10.78069	73.11564	0.15	0.883	-133.0541	154.6154
_cons	341.843	181.9686	1.88	0.061	-16.12976	699.8157

Source	SS	df	MS		
				Number of obs =	339
				F(1, 337) =	6.59
Model	61128.9658	1	61128.9658	Prob > F =	0.0107
Residual	3126229.37	337	9276.645	R-squared =	0.0192
				Adj R-squared =	0.0163
Total	3187358.33	338	9430.05423	Root MSE =	96.315

lgdpgro	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lpopgro	.1292903	.0503661	2.57	0.011	.0302189	.2283618
_cons	280.3133	41.14543	6.81	0.000	199.3791	361.2475

Variable	ols	ols_dum
lpopgro	.12929031*	.05814148
_Icountry_2		4.8024968
_Icountry_3		23.983916
_Icountry_4		-61.154368*
_Icountry_5		-55.759953
_Icountry_6		71.522809**
_Icountry_7		22.472556
_Icountry_8		86.099647
_Icountry_9		-87.803317**
_Icountry_10		10.780687
_cons	280.31333***	341.84296
N	339	339

legend: * p<0.05; ** p<0.01; *** p<0.001

Appendix 3 Panel estimation techniques

```
. xi: xtreg  lgdpgro lpopgro i.year,fe
i.year            _Iyear_1950-2009    (naturally coded; _Iyear_1950 omitted)

Fixed-effects (within) regression               Number of obs      =       339
Group variable: ctry                            Number of groups   =        10

R-sq:  within  = 0.1490                         Obs per group: min =         6
       between = 0.0464                                        avg =      33.9
       overall = 0.0597                                        max =        60

                                                F(60,269)          =      0.79
corr(u_i, Xb)  = -0.7906                        Prob > F           =    0.8691

------------------------------------------------------------------------------
     lgdpgro |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     lpopgro |   .7605937   .4349449     1.75   0.081    -.0957353    1.616923
 _Iyear_1951 |  -40.98947   71.56379    -0.57   0.567    -181.8858    99.90689
 _Iyear_1952 |  -37.99571   71.45078    -0.53   0.595    -178.6696    102.6782
 _Iyear_1953 |  -29.75784   71.34648    -0.42   0.677    -170.2264    110.7107
 _Iyear_1954 |  -41.06829   71.25146    -0.58   0.565    -181.3497    99.21316
 _Iyear_1955 |  -33.02969    71.1641    -0.46   0.643    -173.1391    107.0798
 _Iyear_1956 |  -34.36171   71.08532    -0.48   0.629    -174.3161    105.5926
 _Iyear_1957 |  -22.94429   71.01376    -0.32   0.747    -162.7577    116.8692
 _Iyear_1958 |  -19.70167   70.94973    -0.28   0.781    -159.3891    119.9857
 _Iyear_1959 |  -20.82628   70.89659    -0.29   0.769     -160.409    118.7565
 _Iyear_1960 |  -109.6238    60.4036    -1.81   0.071    -228.5477    9.300167
 _Iyear_1961 |  -87.74264   60.40654    -1.45   0.148    -206.6724    31.18708
 _Iyear_1962 |  -77.87545   60.41447    -1.29   0.198    -196.8208    41.06989
 _Iyear_1963 |   -68.6982   60.42612    -1.14   0.257    -187.6665    50.27006
 _Iyear_1964 |  -66.45111   60.44104    -1.10   0.273    -185.4488    52.54655
 _Iyear_1965 |  -62.68548    60.4597    -1.04   0.301    -181.7199    56.34889
 _Iyear_1966 |  -60.85861   60.48429    -1.01   0.315    -179.9414     58.2242
 _Iyear_1967 |  -54.70754   60.51841    -0.90   0.367    -173.8575    64.44242
 _Iyear_1968 |    -198.34   60.56466    -3.27   0.001     -317.581   -79.09895
 _Iyear_1969 |  -156.2577   60.61089    -2.58   0.010    -275.5898   -36.92568
 _Iyear_1970 |  -145.0668   51.06815    -2.84   0.005    -245.6109    -44.5227
 _Iyear_1971 |  -138.3513    51.1494    -2.70   0.007    -239.0554   -37.64727
 _Iyear_1972 |  -129.4338   51.24072    -2.53   0.012    -230.3177   -28.54999
 _Iyear_1973 |   -122.658   51.32261    -2.39   0.018    -223.7031   -21.61294
 _Iyear_1974 |   -125.865   51.42468    -2.45   0.015     -227.111   -24.61893
 _Iyear_1975 |  -119.0212    51.5398    -2.31   0.022    -220.4939   -17.54848
 _Iyear_1976 |  -110.8254    51.6613    -2.15   0.033    -212.5373   -9.113524
 _Iyear_1977 |   -104.646    51.7932    -2.02   0.044    -206.6176   -2.674423
 _Iyear_1978 |  -96.13875   51.91444    -1.85   0.065     -198.349    6.071541
 _Iyear_1979 |  -93.70237   52.03819    -1.80   0.073    -196.1563    8.751567
 _Iyear_1980 |  -93.30143   52.16077    -1.79   0.075    -195.9967    9.393845
 _Iyear_1981 |  -97.08487   52.29739    -1.86   0.064    -200.0491    5.879381
 _Iyear_1982 |  -97.20503   52.42912    -1.85   0.065    -200.4286    6.018566
 _Iyear_1983 |  -97.62817   52.55625    -1.86   0.064    -201.1021    5.845729
 _Iyear_1984 |  -95.16551   52.68298    -1.81   0.072    -198.8889    8.557902
 _Iyear_1985 |  -92.94244   52.81052    -1.76   0.080    -196.9169    11.03207
 _Iyear_1986 |  -88.78871   52.93538    -1.68   0.095    -193.0091    15.43164
 _Iyear_1987 |  -90.26075   53.06046    -1.70   0.090    -194.7273    14.20585
 _Iyear_1988 |  -86.13444   53.18221    -1.62   0.106    -190.8407    18.57186
 _Iyear_1989 |   -84.9631   53.31231    -1.59   0.112    -189.9255    19.99934
 _Iyear_1990 |  -133.1667   45.76825    -2.91   0.004    -223.2762   -43.05715
 _Iyear_1991 |  -109.3995   45.79388    -2.39   0.018    -199.5595   -19.23946
 _Iyear_1992 |  -115.1622   45.67449    -2.52   0.012    -205.0871   -25.23725
 _Iyear_1993 |  -111.2897   45.56029    -2.44   0.015    -200.9898   -21.58964
 _Iyear_1994 |  -101.2953   45.55359    -2.22   0.027    -190.9822   -11.60843
 _Iyear_1995 |  -91.89233   45.56847    -2.02   0.045    -181.6085   -2.176119
```

```
 _Iyear_1996 |    -80.682   45.56079    -1.77   0.078    -170.3831    9.019093
 _Iyear_1997 |  -79.65478   45.58771    -1.75   0.082    -169.4089    10.09931
 _Iyear_1998 |  -73.52062   45.68832    -1.61   0.109    -163.4728    16.43155
 _Iyear_1999 |  -68.16816   45.75291    -1.49   0.137    -158.2475    21.91118
 _Iyear_2000 |  -63.60586   45.79475    -1.39   0.166    -153.7676    26.55584
 _Iyear_2001 |  -134.7835   47.13355    -2.86   0.005     -227.581   -41.98589
 _Iyear_2002 |  -107.8351   47.17669    -2.29   0.023    -200.7176    -14.9526
 _Iyear_2003 |  -97.18599   45.92017    -2.12   0.035    -187.5946   -6.777339
 _Iyear_2004 |  -90.45919   45.96222    -1.97   0.050    -180.9506    .0322352
 _Iyear_2005 |  -90.43073    45.8519    -1.97   0.050     -180.705   -.1565113
 _Iyear_2006 |  -131.8986   44.79873    -2.94   0.004    -220.0993   -43.69785
 _Iyear_2007 |  -149.4817   44.81625    -3.34   0.001     -237.717   -61.24651
 _Iyear_2008 |  -188.2529   44.82956    -4.20   0.000    -276.5143   -99.99146
 _Iyear_2009 |  -106.2316     44.839    -2.37   0.019    -194.5116   -17.95161
       _cons |  -132.7358   341.1825    -0.39   0.698    -804.4635    538.9918
-------------+----------------------------------------------------------------
     sigma_u |  87.310538
     sigma_e |  89.598029
         rho |   .4870718   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0:     F(9, 269) =     8.73              Prob > F = 0.0000
```

testparm

```
. testparm _Iyear*

 ( 1)  _Iyear_1951 = 0
 ( 2)  _Iyear_1952 = 0
 ( 3)  _Iyear_1953 = 0
 ( 4)  _Iyear_1954 = 0
 ( 5)  _Iyear_1955 = 0
 ( 6)  _Iyear_1956 = 0
 ( 7)  _Iyear_1957 = 0
 ( 8)  _Iyear_1958 = 0
 ( 9)  _Iyear_1959 = 0
 (10)  _Iyear_1960 = 0
 (11)  _Iyear_1961 = 0
 (12)  _Iyear_1962 = 0
 (13)  _Iyear_1963 = 0
 (14)  _Iyear_1964 = 0
 (15)  _Iyear_1965 = 0
 (16)  _Iyear_1966 = 0
 (17)  _Iyear_1967 = 0
 (18)  _Iyear_1968 = 0
 (19)  _Iyear_1969 = 0
 (20)  _Iyear_1970 = 0
 (21)  _Iyear_1971 = 0
 (22)  _Iyear_1972 = 0
 (23)  _Iyear_1973 = 0
 (24)  _Iyear_1974 = 0
 (25)  _Iyear_1975 = 0
 (26)  _Iyear_1976 = 0
 (27)  _Iyear_1977 = 0
 (28)  _Iyear_1978 = 0
 (29)  _Iyear_1979 = 0
 (30)  _Iyear_1980 = 0
 (31)  _Iyear_1981 = 0
 (32)  _Iyear_1982 = 0
 (33)  _Iyear_1983 = 0
 (34)  _Iyear_1984 = 0
 (35)  _Iyear_1985 = 0
 (36)  _Iyear_1986 = 0
 (37)  _Iyear_1987 = 0
 (38)  _Iyear_1988 = 0
 (39)  _Iyear_1989 = 0
 (40)  _Iyear_1990 = 0
 (41)  _Iyear_1991 = 0
```

```
 (42)  _Iyear_1992 = 0
 (43)  _Iyear_1993 = 0
 (44)  _Iyear_1994 = 0
 (45)  _Iyear_1995 = 0
 (46)  _Iyear_1996 = 0
 (47)  _Iyear_1997 = 0
 (48)  _Iyear_1998 = 0
 (49)  _Iyear_1999 = 0
 (50)  _Iyear_2000 = 0
 (51)  _Iyear_2001 = 0
 (52)  _Iyear_2002 = 0
 (53)  _Iyear_2003 = 0
 (54)  _Iyear_2004 = 0
 (55)  _Iyear_2005 = 0
 (56)  _Iyear_2006 = 0
 (57)  _Iyear_2007 = 0
 (58)  _Iyear_2008 = 0
 (59)  _Iyear_2009 = 0

       F( 59,   269) =    0.80
            Prob > F =    0.8507
```

. **We failed to reject the null that all years coefficients are jointly equal to zero therefore no time fixedeffects are needed.**

```
. estimates store fixed

. xi: xtreg  lgdpgro lpopgro i.year,re
i.year            _Iyear_1950-2009    (naturally coded; _Iyear_1950 omitted)

Random-effects GLS regression                   Number of obs      =       339
Group variable: ctry                            Number of groups   =        10

R-sq:  within  = 0.1451                         Obs per group: min =         6
       between = 0.0292                                        avg =      33.9
       overall = 0.1063                                        max =        60

Random effects u_i ~ Gaussian                   Wald chi2(60)      =     45.80
corr(u_i, X)       = 0 (assumed)                Prob > chi2        =    0.9120

------------------------------------------------------------------------------
     lgdpgro |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     lpopgro |   .2798707   .2033972     1.38   0.169    -.1187805    .6785219
 _Iyear_1951 |  -56.28473   70.55534    -0.80   0.425    -194.5707    82.00118
 _Iyear_1952 |  -52.39935   70.53754    -0.74   0.458    -190.6504    85.85168
 _Iyear_1953 |   -43.2677   70.52172    -0.61   0.540    -181.4877    94.95233
 _Iyear_1954 |  -53.68698   70.50796    -0.76   0.446      -191.88    84.50609
 _Iyear_1955 |  -44.74231   70.49604    -0.63   0.526     -182.912    93.42739
 _Iyear_1956 |  -45.15891   70.48611    -0.64   0.522    -183.3091    92.99132
 _Iyear_1957 |  -32.79237   70.47806    -0.47   0.642    -170.9268    105.3421
 _Iyear_1958 |  -28.55334   70.47207    -0.41   0.685    -166.6761    109.5694
 _Iyear_1959 |  -28.67037   70.46858    -0.41   0.684    -166.7862    109.4455
 _Iyear_1960 |  -112.9651   60.12139    -1.88   0.060    -230.8009    4.870631
 _Iyear_1961 |  -90.35182   60.12901    -1.50   0.133    -208.2025    27.49888
 _Iyear_1962 |  -79.87784   60.13654    -1.33   0.184    -197.7433    37.98761
 _Iyear_1963 |  -70.14497   60.14439    -1.17   0.244    -188.0258    47.73587
 _Iyear_1964 |  -67.37024    60.1527    -1.12   0.263    -185.2674    50.52689
 _Iyear_1965 |    -63.078   60.16182    -1.05   0.294     -180.993      54.837
 _Iyear_1966 |  -60.67713   60.17269    -1.01   0.313    -178.6134    57.25918
 _Iyear_1967 |  -53.86012   60.18654    -0.89   0.371    -171.8236    64.10332
 _Iyear_1968 |  -196.7322   60.20395    -3.27   0.001    -314.7298   -78.73463
 _Iyear_1969 |  -153.9929   60.22038    -2.56   0.011    -272.0227   -35.96313
 _Iyear_1970 |  -139.9699   50.51022    -2.77   0.006    -238.9681    -40.9717
 _Iyear_1971 |  -132.6094   50.53302    -2.62   0.009    -231.6523   -33.56648
```

```
 _Iyear_1972 |  -123.0217   50.55826    -2.43   0.015     -222.114   -23.92932
 _Iyear_1973 |  -115.6844   50.58061    -2.29   0.022    -214.8206   -16.54824
 _Iyear_1974 |  -118.2342   50.60818    -2.34   0.019    -217.4244   -19.04395
 _Iyear_1975 |  -110.6957   50.63897    -2.19   0.029    -209.9463   -11.44513
 _Iyear_1976 |  -101.8109   50.67118    -2.01   0.045    -201.1246   -2.497197
 _Iyear_1977 |  -94.92584   50.70588    -1.87   0.061    -194.3075    4.455856
 _Iyear_1978 |  -85.80285   50.73757    -1.69   0.091    -185.2467    13.64096
 _Iyear_1979 |  -82.76576   50.76976    -1.63   0.103    -182.2727    16.74113
 _Iyear_1980 |  -81.79398    50.8015    -1.61   0.107    -181.3631    17.77514
 _Iyear_1981 |  -84.96605   50.83676    -1.67   0.095    -184.6043    14.67216
 _Iyear_1982 |  -84.51868   50.87063    -1.66   0.097    -184.2233    15.18593
 _Iyear_1983 |  -84.41229   50.90325    -1.66   0.097    -184.1808    15.35623
 _Iyear_1984 |  -81.43782   50.93568    -1.60   0.110    -181.2699    18.39429
 _Iyear_1985 |  -78.71435   50.96827    -1.54   0.122    -178.6103    21.18163
 _Iyear_1986 |  -74.08371   51.00012    -1.45   0.146    -174.0421    25.87469
 _Iyear_1987 |   -75.0899   51.03199    -1.47   0.141    -175.1108    24.93096
 _Iyear_1988 |  -70.52065   51.06297    -1.38   0.167    -170.6022    29.56093
 _Iyear_1989 |  -68.88661   51.09605    -1.35   0.178     -169.033    31.25982
 _Iyear_1990 |  -116.5801   43.00243    -2.71   0.007    -200.8633   -32.29684
 _Iyear_1991 |   -92.7368   43.00835    -2.16   0.031    -177.0316   -8.441991
 _Iyear_1992 |  -98.85596   42.98083    -2.30   0.021    -183.0968   -14.61508
 _Iyear_1993 |  -95.33006   42.95457    -2.22   0.026    -179.5195   -11.14065
 _Iyear_1994 |  -85.35618   42.95303    -1.99   0.047    -169.5426   -1.169792
 _Iyear_1995 |  -75.90763   42.95645    -1.77   0.077    -160.1007    8.285464
 _Iyear_1996 |  -64.72078   42.95468    -1.51   0.132    -148.9104    19.46886
 _Iyear_1997 |  -63.61137   42.96087    -1.48   0.139    -147.8131    20.59039
 _Iyear_1998 |  -57.17279   42.98402    -1.33   0.183    -141.4199    27.07433
 _Iyear_1999 |  -51.62716    42.9989    -1.20   0.230    -135.9034    32.64913
 _Iyear_2000 |  -46.94064   43.00855    -1.09   0.275    -131.2358    37.35456
 _Iyear_2001 |  -117.3597   44.41108    -2.64   0.008    -204.4038   -30.31559
 _Iyear_2002 |   -90.2815   44.42131    -2.03   0.042    -177.3457   -3.217338
 _Iyear_2003 |   -80.1525   43.03751    -1.86   0.063    -164.5045    4.199475
 _Iyear_2004 |   -73.3036   43.04724    -1.70   0.089    -157.6746    11.06743
 _Iyear_2005 |  -70.34215   43.00249    -1.64   0.102    -154.6255    13.94118
 _Iyear_2006 |  -112.5712   41.85031    -2.69   0.007    -194.5963   -30.54614
 _Iyear_2007 |  -130.1051    41.8544    -3.11   0.002    -212.1383   -48.07203
 _Iyear_2008 |  -168.8389   41.85751    -4.03   0.000    -250.8782   -86.79974
 _Iyear_2009 |  -86.79124   41.85971    -2.07   0.038    -168.8348   -4.747705
       _cons |   256.9051   155.7634     1.65   0.099    -48.38564    562.1958
-------------+----------------------------------------------------------------
     sigma_u |  71.607679
     sigma_e |  89.598029
         rho |  .38977407   (fraction of variance due to u_i)
------------------------------------------------------------------------------

.
```

```
. estimates table fixed random, star stats(N r2 r2_a)

----------------------------------------------
    Variable |     fixed          random      
-------------+--------------------------------
     lpopgro |   .7605937       .27987068     
 _Iyear_1951 | -40.989471      -56.284735     
 _Iyear_1952 | -37.995715       -52.39935     
 _Iyear_1953 | -29.757835      -43.267699     
 _Iyear_1954 | -41.068291       -53.68698     
 _Iyear_1955 | -33.029687      -44.742312     
 _Iyear_1956 | -34.361712      -45.158912     
 _Iyear_1957 | -22.944289      -32.792366     
 _Iyear_1958 | -19.701667      -28.553338     
 _Iyear_1959 |  -20.82628      -28.670366     
 _Iyear_1960 | -109.62376      -112.96512     
 _Iyear_1961 | -87.742636      -90.351818     
 _Iyear_1962 | -77.875454      -79.877844     
 _Iyear_1963 | -68.698204      -70.144973     
```

```
  _Iyear_1964 | -66.451109      -67.370239   
  _Iyear_1965 | -62.685482         -63.078   
  _Iyear_1966 | -60.858608      -60.677127   
  _Iyear_1967 | -54.707543      -53.860119   
  _Iyear_1968 | -198.33999**     -196.7322** 
  _Iyear_1969 | -156.25773*      -153.9929*  
  _Iyear_1970 |  -145.0668**    -139.96991** 
  _Iyear_1971 | -138.35133**    -132.60937** 
  _Iyear_1972 | -129.43385*     -123.02167*  
  _Iyear_1973 | -122.65802*     -115.68442*  
  _Iyear_1974 | -125.86497*     -118.23417*  
  _Iyear_1975 | -119.02118*     -110.69569*  
  _Iyear_1976 | -110.82543*     -101.81088*  
  _Iyear_1977 | -104.64602*     -94.925836   
  _Iyear_1978 | -96.138746      -85.802845   
  _Iyear_1979 | -93.702372      -82.765761   
  _Iyear_1980 | -93.301426       -81.79398   
  _Iyear_1981 | -97.084873      -84.966048   
  _Iyear_1982 | -97.205033      -84.518683   
  _Iyear_1983 | -97.628174      -84.412295   
  _Iyear_1984 | -95.165505      -81.437819   
  _Iyear_1985 | -92.942442      -78.714345   
  _Iyear_1986 | -88.788709      -74.083709   
  _Iyear_1987 | -90.260748      -75.089896   
  _Iyear_1988 | -86.134437      -70.520653   
  _Iyear_1989 | -84.963103      -68.886611   
  _Iyear_1990 | -133.16668**    -116.58006** 
  _Iyear_1991 | -109.39946*     -92.736801*  
  _Iyear_1992 | -115.16219*     -98.855958*  
  _Iyear_1993 | -111.28974*      -95.33006*  
  _Iyear_1994 | -101.29533*     -85.356181*  
  _Iyear_1995 | -91.892333*     -75.907629   
  _Iyear_1996 |    -80.682      -64.720779   
  _Iyear_1997 | -79.654784      -63.611366   
  _Iyear_1998 | -73.520622      -57.172791   
  _Iyear_1999 | -68.168159       -51.62716   
  _Iyear_2000 | -63.605863      -46.940641   
  _Iyear_2001 | -134.78347**    -117.35971** 
  _Iyear_2002 |  -107.8351*     -90.281499*  
  _Iyear_2003 | -97.185988*     -80.152504   
  _Iyear_2004 | -90.459194      -73.303605   
  _Iyear_2005 | -90.430732*     -70.342153   
  _Iyear_2006 | -131.89859**    -112.57124** 
  _Iyear_2007 | -149.48174***   -130.10514** 
  _Iyear_2008 | -188.25289***   -168.83895***
  _Iyear_2009 | -106.23162*     -86.791237*  
        _cons | -132.73585        256.9051   
-------------+--------------------------------
            N |        339             339   
           r2 |  .14902846                   
         r2_a | -.06925048                   
----------------------------------------------
      legend: * p<0.05; ** p<0.01; *** p<0.001

.
```

Hausman test

```
. hausman fixed random

                 ---- Coefficients ----
             |      (b)          (B)            (b-B)     sqrt(diag(V_b-V_B))
             |     fixed        random       Difference          S.E.
-------------+----------------------------------------------------------------
     lpopgro |    .7605937     .2798707         .480723        .3844562
 _Iyear_1951 |   -40.98947    -56.28473        15.29526        11.97167
 _Iyear_1952 |   -37.99571    -52.39935        14.40363        11.38728
```

_Iyear_1953	-29.75784	-43.2677	13.50986	10.81699
_Iyear_1954	-41.06829	-53.68698	12.61869	10.26638
_Iyear_1955	-33.02969	-44.74231	11.71262	9.728177
_Iyear_1956	-34.36171	-45.15891	10.7972	9.210406
_Iyear_1957	-22.94429	-32.79237	9.848077	8.706126
_Iyear_1958	-19.70167	-28.55334	8.851671	8.21897
_Iyear_1959	-20.82628	-28.67037	7.844086	7.778513
_Iyear_1960	-109.6238	-112.9651	3.341357	5.832114
_Iyear_1961	-87.74264	-90.35182	2.609181	5.783722
_Iyear_1962	-77.87545	-79.87784	2.00239	5.788372
_Iyear_1963	-68.6982	-70.14497	1.446769	5.828173
_Iyear_1964	-66.45111	-67.37024	.9191297	5.896799
_Iyear_1965	-62.68548	-63.078	.3925173	5.994197
_Iyear_1966	-60.85861	-60.67713	-.1814807	6.131626
_Iyear_1967	-54.70754	-53.86012	-.8474237	6.329167
_Iyear_1968	-198.34	-196.7322	-1.607786	6.600189
_Iyear_1969	-156.2577	-153.9929	-2.264839	6.869222
_Iyear_1970	-145.0668	-139.9699	-5.096894	7.528222
_Iyear_1971	-138.3513	-132.6094	-5.741962	7.916782
_Iyear_1972	-129.4338	-123.0217	-6.412175	8.335137
_Iyear_1973	-122.658	-115.6844	-6.973604	8.69553
_Iyear_1974	-125.865	-118.2342	-7.630801	9.127388
_Iyear_1975	-119.0212	-110.6957	-8.325484	9.594054
_Iyear_1976	-110.8254	-101.8109	-9.014546	10.06587
_Iyear_1977	-104.646	-94.92584	-9.720186	10.55699
_Iyear_1978	-96.13875	-85.80285	-10.3359	10.99127
_Iyear_1979	-93.70237	-82.76576	-10.93661	11.41952
_Iyear_1980	-93.30143	-81.79398	-11.50745	11.83018
_Iyear_1981	-97.08487	-84.96605	-12.11882	12.27361
_Iyear_1982	-97.20503	-84.51868	-12.68635	12.68822
_Iyear_1983	-97.62817	-84.41229	-13.21588	13.07743
_Iyear_1984	-95.16551	-81.43782	-13.72769	13.45557
_Iyear_1985	-92.94244	-78.71435	-14.2281	13.82702
_Iyear_1986	-88.78871	-74.08371	-14.705	14.18247
_Iyear_1987	-90.26075	-75.0899	-15.17085	14.53095
_Iyear_1988	-86.13444	-70.52065	-15.61378	14.86337
_Iyear_1989	-84.9631	-68.88661	-16.07649	15.21169
_Iyear_1990	-133.1667	-116.5801	-16.58663	15.66919
_Iyear_1991	-109.3995	-92.7368	-16.66266	15.72775
_Iyear_1992	-115.1622	-98.85596	-16.30623	15.45338
_Iyear_1993	-111.2897	-95.33006	-15.95968	15.187
_Iyear_1994	-101.2953	-85.35618	-15.93915	15.17124
_Iyear_1995	-91.89233	-75.90763	-15.9847	15.20623
_Iyear_1996	-80.682	-64.72078	-15.96122	15.18819
_Iyear_1997	-79.65478	-63.61137	-16.04342	15.25134
_Iyear_1998	-73.52062	-57.17279	-16.34783	15.48539
_Iyear_1999	-68.16816	-51.62716	-16.541	15.63405
_Iyear_2000	-63.60586	-46.94064	-16.66522	15.72972
_Iyear_2001	-134.7835	-117.3597	-17.42376	15.78695
_Iyear_2002	-107.8351	-90.2815	-17.5536	15.88671
_Iyear_2003	-97.18599	-80.1525	-17.03348	16.01358
_Iyear_2004	-90.45919	-73.3036	-17.15559	16.10779
_Iyear_2005	-90.43073	-70.34215	-20.08858	15.9117
_Iyear_2006	-131.8986	-112.5712	-19.32735	15.98369
_Iyear_2007	-149.4817	-130.1051	-19.3766	16.02204
_Iyear_2008	-188.2529	-168.8389	-19.41394	16.05112
_Iyear_2009	-106.2316	-86.79124	-19.44038	16.07172

```
                           b = consistent under Ho and Ha; obtained from xtreg
            B = inconsistent under Ha, efficient under Ho; obtained from xtreg

    Test:  Ho:  difference in coefficients not systematic

                  chi2(60) = (b-B)'[(V_b-V_B)^(-1)](b-B)
                          =        2.92
                Prob>chi2 =      1.0000
```

Appendix 4 Unit root tests

```
. xtunitroot fisher  gdpgro, dfuller trend lags(4)
(1 missing value generated)

Fisher-type unit-root test for gdpgro
Based on augmented Dickey-Fuller tests
--------------------------------------
Ho: All panels contain unit roots           Number of panels       =     10
Ha: At least one panel is stationary        Avg. number of periods =  59.90

AR parameter: Panel-specific                Asymptotics: T -> Infinity
Panel means:  Included
Time trend:   Included
Drift term:   Not included                  ADF regressions: 4 lags
------------------------------------------------------------------------------
                                  Statistic      p-value
------------------------------------------------------------------------------
 Inverse chi-squared(20)   P        49.1548       0.0003
 Inverse normal            Z        -3.8714       0.0001
 Inverse logit t(49)       L*       -4.0690       0.0001
 Modified inv. chi-squared Pm        4.6098       0.0000
------------------------------------------------------------------------------
 P statistic requires number of panels to be finite.
 Other statistics are suitable for finite or infinite number of panels.
------------------------------------------------------------------------------
xtunitroot fisher  popgro, dfuller trend lags(4)
(1 missing value generated)

Fisher-type unit-root test for popgro
Based on augmented Dickey-Fuller tests
--------------------------------------
Ho: All panels contain unit roots           Number of panels       =     10
Ha: At least one panel is stationary        Avg. number of periods =  59.90

AR parameter: Panel-specific                Asymptotics: T -> Infinity
Panel means:  Included
Time trend:   Included
Drift term:   Not included                  ADF regressions: 4 lags
------------------------------------------------------------------------------
                                  Statistic      p-value
------------------------------------------------------------------------------
 Inverse chi-squared(20)   P        61.3497       0.0000
 Inverse normal            Z        -4.5153       0.0000
 Inverse logit t(54)       L*       -5.0274       0.0000
 Modified inv. chi-squared Pm        6.5380       0.0000
------------------------------------------------------------------------------
 P statistic requires number of panels to be finite.
 Other statistics are suitable for finite or infinite number of panels.
----------------------------------------------------------------------------
```

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