STOCHASTIC MODELS AND TIME SERIES

Application of economic-mathematical models for assessment of unemployment of young people

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Abstract

The unemployment is a common problem of modern labor markets and developed economies. Neither country is immune to the problem of unemployment, but they all have different unemployment rates. The most affected category that most felt the problem of unemployment is the young population. Reducing the rate of unemployed young people is a long and difficult process. The Republic of Macedonia is a country where the unemployment rate of young people is high. In this paper is analyzed the labor market in the country and the data are collected from the Nacional Institute of Statistics and Employment Agency about the number of unemployed youth in the past 10 years. Based on the data it is made estimation and forecasting for the unemployment trend of young people in Republic of Macedonia in the period from 2004 to 2013. Linear and exponential trend are the models which are used for the analysis. These two models are implemented to gain better insight into the trend of young unemployment in the next years.

Keywords: young unemployment, linear trend, exponential trend

1. Introduction

The unemployment is a worldwide problem, especially expressed in the developing countries, including Republic of Macedonia. The young unemployment has always been a major problem in Europe, because of the bigger unemployment rates compared with the USA. As the main problem for the high rate of young unemployment is potentiated the mismatches between education and the needs of the labor market. In Republic of Macedonia, the demand for high educated labor force is not compatible with the supply of the labor market. For making the real and right decisions we need to forecast the direction of movement of unemployment. The forecast of the unemployment rate is very important for the economists to create strategies and decision making in order to assist in developing in the economy in the future.

The purpose of this paper is predicting the number of unemployed young people in the next ten years based on the given data, i.e. to determine the direction and the tendency of movement of youth unemployment in the country. For this purpose, the data refer to the number of young unemployment people in the period from 2004 to 2013, based on this data we predict the trend of unemployment for the next ten years. Namely, the data which are analyzed were taken from the National Institute of Statistics and the Employment Agency of the Republic of Macedonia. Application of linear and exponential trend model identify the tendency of increasing or decreasing the number of youth unemployed in Republic of Macedonia.

2. Mathematical model

2.1 Linear trend model

Linear regression lives up to its name: it's a very straightforward approach for predicting a quantitative response Y on the basis of a single predictor variable X. It assumes that there is approximately a linear relationship between X and Y. Mathematically, we can write this linear relationship as:

$$Y \approx \beta_0 + \beta_1 X \qquad (1)$$

You might read "≈" as "approximately modeled as". We will sometimes describe (1) by saying that we are regressing Y on X (or Y onto X).

In equation (1), β_0 and β_1 are two unknown constants that represent the *intercept* and *slope* terms in the linear model. Together, β_0 and β_1 are known as the model *coefficients* or *parameters*. We will produce estimates β_0 and β_1 for the model coefficients

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x \qquad (2)$$

where $\hat{y}$ indicates a prediction of Y on the basis of $X = x$. Here we use hat symbol, $\wedge$, to denote the estimated value for an unknown parameter or coefficient, or to denote the predicted value of the response.

In practice, β_0 and β_1 are unknown. So before we can use (1) to make predictions, we must use data to estimate the coefficients. Let

$$(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n) \qquad (3)$$

represent n observation pairs, each of which consists of a measurement of X and a measurement of Y. The goal is to obtain coefficient estimates β_0 and β_1 such that the linear model (1) fits the available data well, so that $y_i \approx \hat{\beta}_0 + \hat{\beta}_1 x_i$ for $i = 1, \cdots, n$. In other words, we want to find an intercept β_0 and a slope β_1 such that the resulting line is as close as possible. There are a number of ways of measuring the closeness. However, by far the most common approach involves minimizing the least squares criterion.

Let $\hat{y}_i \approx \hat{\beta}_0 + \hat{\beta}_1 x_i$ be the prediction for Y based on the i th value of X. Then $e_i = y_i - \hat{y}_i$ represents the ith *residual* – this is the difference between the ith observed response value and the ith response value that is predicted by the linear model. We define the *residual sum squares* (RSS) as

$$RSS = e_1^2 + e_2^2 + \cdots + e_n^2 \qquad (4)$$

Or equivalently as

$$RSS = \left(y_1 - \hat{\beta}_0 - \hat{\beta}_1 x_1\right)^2 + \left(y_2 - \hat{\beta}_0 - \hat{\beta}_1 x_2\right)^2 + \cdots + \left(y_n - \hat{\beta}_0 - \hat{\beta}_1 x_n\right)^2 \qquad (5)$$

The least squares approach chooses β_0 and β_1 to minimize RSS. Using some calculus, one can show that the minimizers are

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \qquad (6)$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} \qquad (7)$$

Where $\bar{y} \equiv \frac{1}{n}\sum_{i=1}^{n} y_i$ and $\bar{x} \equiv \frac{1}{n}\sum_{i=1}^{n} x_i$ ae the sample means. In other words, (6) and (7) defines the *least squares coefficient estimates* for linear regression.

2.2. Exponential trend model

When a time series increases at a rate such that the percentage difference from value to value is constant, an exponential trend is present. Equation (8) defines the exponential trend model.

$$Y_i = \beta_0 \beta_1^{X_i} \varepsilon_i \quad (8)$$

Where $\beta_0 = Y$ intercept, $(\beta_1 - 1) \times 100\%$ is the annual compound growth rate (in %).

The model in equation (8) is not in the form of a linear regression model. To transform this nonlinear model to a linear model we use a base 10 logarithm transformation. Taking the logarithm of each site of equation (8) results in equation (9).

$$\log(Y_i) = \log(\beta_0 \beta_1^{X_i} \varepsilon_i) = \log(\beta_0) + (\beta_1^{X_i}) + \log(\varepsilon_i) = \log(\beta_0) + X_i \log(\beta_1) + \log(\varepsilon_i) \quad (9)$$

Equation (9) is a linear model we can estimate using the least – squares method, with log (Y_i) as the dependent variable and X_i as the independent variable. This result in equation (10).

$$\log(\hat{Y}_i) = b_0 + b_1 X_i \quad (10a)$$

Where,

$b_0 =$ estimate of $\log(\beta_0)$ and thus $10^{b_0} = \hat{\beta}_0$

$b_1 =$ estimate of $\log(\beta_1)$ and thus $10^{b_1} = \hat{\beta}_1$

Therefore,

$$\hat{Y}_i = \hat{\beta}_0 \hat{\beta}_1^{X_i} \quad (10b)$$

Where, $(\hat{\beta}_1 - 1) \times 100\%$ is the estimated annual compound growth rate (in %).

3. Results

3.1. Results of linear trend model

The Table 1 shows the movement of the number of unemployed in period from 2004 to 2013.

Year	(X_t)	(Y_t)
2004	0	125418
2005	1	121495
2006	2	122561
2007	3	118867
2008	4	115906
2009	5	109776
2010	6	108380
2011	7	108627
2012	8	108262
2013	9	103210
Total	45	1142502

Table 1. Movement of unemployment in period from 2004 to 2013

From the Table 1 it can be noted that the unit for X is one year and the unit for Y are thousand unemployed people.

$\bar{X} = 4{,}5$; $\bar{Y} = 114250{,}2$;

$\hat{\beta}_1 = -2422{,}8$; $\hat{\beta}_0 = 125152{,}8$

The coefficient β_1 indicates that the number of unemployed in Republic of Macedonia, in the analyzed period, declines for 2.423 unemployed, and the coefficient β_0 is expected trend value and is 125.153 youth unemployed.

The estimated model of linear trend is:

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X$$

$$\hat{Y} = -2422{,}8X + 125152{,}8$$

The variance of the trend is $\sigma^2_{\hat{Y}} = 2990133{,}4$.

The standard deviation of the trend is $\sigma_{\hat{Y}} = 5468{,}21$

The representativeness of the trend is measured by standard deviation (standard error).

The coefficient of variation of the trend shows that the percentage of standard deviation from the mean trend variable Y is 4,79% (less than 10%) and suggest a good representation of the estimated linear trend model: $V_{\hat{Y}} = 4{,}79$

The prediction of the movement of the number of the unemployed in the period from 2015 to 2024 is shown in Table 2.

Year	(X_t)	(Y_t)
2015	10	100925
2016	11	98573
2017	12	95866

2018	13	93890
2019	14	91918
2020	15	90032
2021	16	87357
2022	17	84556
2023	18	82101
2024	19	80173

Table 2. Movement of the number of unemployment in period from 2015 to 2024

To determine the number of unemployed for each year in the future, starting from 2015 to 2024, is necessary to include the period X in the linear function.

$$\hat{Y} = -2422{,}8X + 125152{,}8$$

From the Figure 1 we can see that the number of unemployed will decline linearly in the next 10 years, so for 2015 assumes that the number of unemployed will be 100.925 and this trend will continue and in 2024 is expected 80.173 unemployed people.

The possibility of achieving forecast, i.e. prediction for 2024 is 94,19% and this is seen by the coefficient of determination which is $R^2 = 0{,}9419$.

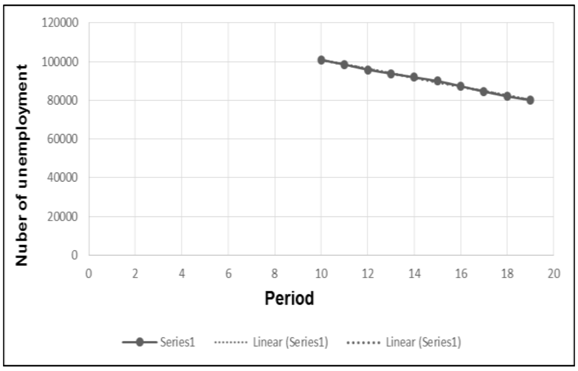

Figure 1. Function of linear trend in period from 2015 to 2024

Figure 2 shows the whole observed period from 2004 to 2024 using linear trend model.

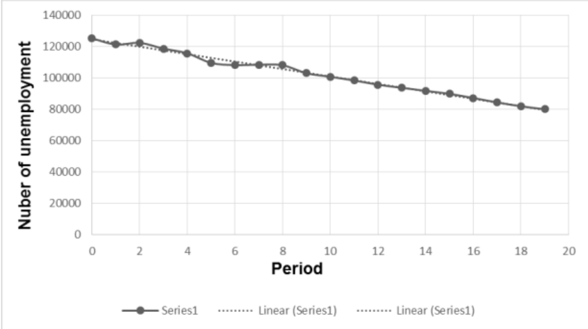

Figure 2. Function of linear trend in period from 2004 to 2024.

We can conclude that with the application of the linear trend model there will be a downward trend, i.e. linear decline of the number of unemployed in the next 10 years.

3.2 Results of exponential trend model

Below are shown the results of using the exponential trend model. The data for number of unemployment young people are same as Table 1.

The model of the exponential trend is:

$$\hat{Y} = \hat{\beta}_0 \cdot \hat{\beta}_1^X$$

To evaluate the parameters of the model with linear squares method is necessary to use logarithm:

$$\log \hat{Y} = \log \hat{\beta}_0 + \log \hat{\beta}_1 \cdot X$$

The evaluation of the parameters is:

$\bar{X} = 4{,}5;\ \overline{\log Y} = 5{,}057003;$

$\log \hat{\beta}_1 = -0{,}009202; \log \hat{\beta}_0 = 5{,}098412$

The estimated logarithmic model is:

$$\log \bar{Y} = 5{,}098412 - 0{,}009202X$$

With transformation:

$$\hat{\beta}_0 = 125433{,}05; \hat{\beta}_1 = 0{,}97903$$

We get the values of the parameters of the model, therefore the estimated model of the exponential trend is:

$$\hat{Y} = 125433{,}05 \cdot 0{,}97903^X$$

The parameter $\hat{\beta}_1$ shows estimation of the average rate of change of the variable Y per unit time X.

$$\bar{S} = -2{,}097\%$$

This means that the number of unemployed declines to 2,097% per year.

Representativeness of the model is examined with the coefficient of variation:

$SR = 0{,}000426;\ \hat{\sigma}_{\log \hat{Y}} = 0{,}00729;$

$\hat{V}_{\log \hat{Y}} = 0{,}1442$

The coefficient of variation is 0,14% (less than 10%) and shows the good representation of exponential trend model.

The assessment of the movement of the number of unemployed in the period from 2015 to 2024 is shown in Table 3.

Year	(X_t)	(Y_t)
2015	10	125433
2016	11	119082
2017	12	115368
2018	13	111772
2019	14	109766
2020	15	108810
2021	16	110045
2022	17	111629
2023	18	112991
2024	19	114492

Table 3. Movement of the number of unemployment in period from 2015 to 2024

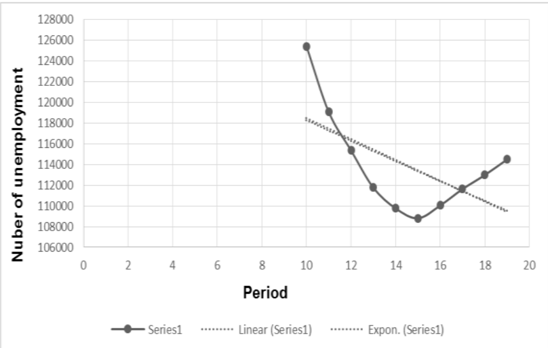

Figure 3. Function of exponential trend in period from 2015 to 2024

Exponential trend shows that after the fall of unemployment in 2013 are noticed its increase in 2015. Then will continue to decline and reach its minimum for which is expected to again start increasing.

The coefficient of determination is $R^2 = 0{,}9425$.

In Figure 4 is represented the entire period from 2004 to 2024 using an exponential trend.

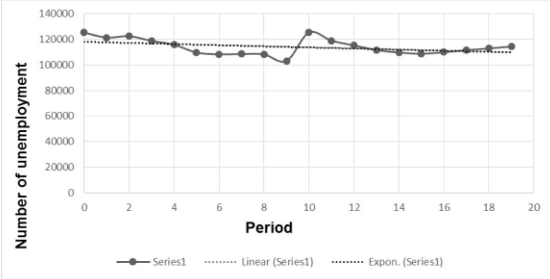

Figure 4. Function of exponential trend in period from 2004 to 2024

According to this, we can conclude that with application of exponential trend there will be a trend of exponential decline of the number of unemployed in the next 10 years.

4. Conclusion

This paper shows a way to predict the unemployment of young people in Republic of Macedonia in the next 10 years. For this purpose were used linear and exponential trend models. According to the obtained data we can conclude that there is expected downward trend of youth unemployment in Republic of Macedonia.

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