MEASURING POVERTY USING PARAMETERIZED LORENZ CURVE – THE CASE OF MACEDONIA

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ABSTRACT

The primary aim of this paper is to show how can we construct poverty measures from grouped data, i.e., to show how can we derive poverty measures from parameterized Lorenz curve? In this paper, Gaurav Datt's approach has been applied. The derived poverty measures are estimated in the case of Macedonia, using interactive software package "Povcal", created by the World Bank.

Our findings suggest two main conclusions: 1) the high poverty level is accompanied with a moderate level of income inequality, and 2) the transmission mechanism from the economic growth to poverty reduction is working properly.

Key words: parameterized Lorenz curve, general quadratic Lorenz curve, Foster - Greer - Thorbecke index.

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1. INTRODUCTION

The grouped data are the most common form of information available to researchers, when it comes to the problem of poverty and income distribution.

In general, there are two basic approaches when constructing poverty measures from grouped data: the interpolation methods [1], and the methods based on parameterized Lorenz curve. In this paper, the Gaurav Datt's approach, which is based on a parameterized Lorenz curve [2] with a General Quadratic functional form, has been used.

In accordance with our knowledge, this is the first attempt in Macedonia using a parameterized Lorenz curve: 1) to construct poverty measures of a so-called P-alfa class of measures, and 2) to calculate poverty measures' elasticises with respect to the mean income and the Gini index.

The paper structure is as follows: in the first part, some relevant papers and studies related with our paper are presented. The second part explains the methodological background and sources of data. In the third part are being sublimated the obtained results, while the fourth part concludes.

2. LITERATURE REVIEW

In the literature not to many attempts are being made to test the theoretical validity and empirical performance of the alternative functional forms of the Lorenz curve [6]. First, Kakwani [3] set the mathematical formulation for parameterization of the Beta Lorenz curve, and later Villasenor and Arnold [4] did the same for the General Quadratic (GQ) Lorenz curve.

In his seminal paper, using Foster-Greer-Torbeke (FGT) class of poverty measures, Datt [2] has showed how to construct poverty measures when grouped data are available. In the same paper, he has explained the means of constructing point estimates of the elasticities of poverty measures with respect to the mean income and the Gini index. To estimate the Lorenz curve, he has used the mathematical formulations (functional forms of the GQ and Beta Lorenz curve) proposed by Kakwani and Villasenor and Arnold.

Essama-Nssah [5], following the procedure proposed by Datt, uses regression analysis to fit the data to a model such as the General Quadratic model. In fact, Essama-Nssah's simulation strategy is a modification of Datt's approach. For a parameterization of the Lorenz curve, he computes the associated first and second order derivatives. Then, he combines these results with an estimate of the mean of the distribution to recover levels of the welfare indicator (using the first order derivative) along with an estimate of the density function (based on the second order derivative).

Minoiu and Reddy [6] asses the performance of functional forms proposed by Kakwani and Villasenor and Arnold to estimate the Lorenz curve from grouped data. The methods are implemented using the computational tools such as Povcal and SimSIP, both developed by the World Bank. To identify biases associated with these methods, they use unit data from several household surveys and theoretical distributions. They are concluding that poverty and inequality are better estimated when the true distribution is unimodal than multimodal.

More comprehensive poverty and inequality studies, based on parameterized Lorenz curve, for a different functional forms, can be find in: Bhalla [7]; Chen and Ravallion [8]; Figini and Santarelli, [9]; Pritchett [10]; Son and Kakwani [11]; Kamin [12]; Edward and Sumner [13]; Kakwani and Podder [14], [15].

3. METHODOLOGY AND DATA

The used methodology is based on the following two functions [2]:

1) Lorenz curve:

$$L = L(p;\pi) \qquad (1)$$

where L is the share of the aggregate income that belongs to the poorest p percentages of the households, and π is a vector of the Lorenz curve (estimable) parameters.

2) Poverty measure:

$$P = P(\mu / z;\pi) \qquad (2)$$

where P is a poverty measure given as a function of the coefficient of the mean income μ and the poverty line z, and the parameters of the Lorenz curve π.

The function L covers relative inequalities in the households and supports alternative parameterizations of the Lorenz curve, while the function P, which is homogenous of degree zero in mean income and poverty line[3], covers the assessment of the absolute living standard of the poor households, and supports different poverty measures [2].

Regarding the poverty measures, we use the FGT index:

$$P_\alpha = \int_o^z \left[\frac{z-x}{z}\right]^\alpha f(x)dx \quad \alpha \geq 0 \qquad (3)$$

where x is the household income; $f(x)$ is its density (roughly estimated proportion of households with income x); z is poverty line, and α is non-negative parameter[4]. We prefer this index since it belongs to the class of additively separable indicies[5]. It incorporates: head-count index *(H)*; poverty gap index *(PG)*; and poverty severity index *(PS)*, where: H corresponds to $\alpha = 0$; PG to $\alpha = 1$, while PS corresponds to $\alpha = 2$ (see eq.3).

Given the best performances, the estimation of the Lorenz curve is usually based on the following two functional forms: GQ Lorenz curve [4] and Beta Lorenz curve [3]. In this paper our focus is aimed at the GQ Lorenz curve, which specification, as well as the equations for estimation of the poverty measures (H, PG, PS), are given in Annex 1, Table A1.1.

Therefore, in order to estimate the poverty measures, in the first step, we estimate the parameters of the GQ Lorenz curve, by using the following regression:

$$L(1-L) = a(p^2 - L) + bL(p-1) + c(p-L) \tag{4}$$

The regression (4) does not contain an intercept. The parameters are estimated with the OLS method, using all except the last observation for *(p, L)*. The last observation that takes values (1, 1) is excluded since the functional form for the Lorenz curve already is being established to pass through the points (1, 1). Then, in the second step: 1) we compute the mean income μ, 2) we set the poverty line z, and 3) we check out whether the parameterization enables theoretical validity of the Lorenz curve (for the conditions of theoretical validity of the Lorenz curve, see: Annex 1, Table A1.2). Finally, in the third step, we construct point estimates of the elasticities of poverty measures with respect to the mean income ant the Gini index (the formulas are given in Annex 1, Table A1.3).

The grouped data at monthly frequency have been obtained from the study “Material deprivation poverty and social exclusion in Republic of Macedonia” [16] (see: Annex 2, Table A2.1).

4. RESULTS

The estimated parameters a, *b* and c of the general quadratic Lorenz curve are presented in Annex 2, Table A2.2. Our Lorenz curve satisfies previously outlined theoretical conditions regarding its validity.

The Gini index equals 37.84, which implies moderate -, to high level of inequality in income distribution among the households (see: Annex 2, Table A2.3 and Figure A2.1).

The mean income μ is equal to 19073,70 denars (or about 347,00 US$), while the poverty line z is set to a 60% of the mean income, which is 11444,00 denars (or about 208,00 US$), (see: Annex 2, Table A2.3).

The estimated head-count index *(H)* proves that 33.38% of the total number of households are below the poverty line (see: Annex 2, Table A2.3).

The estimated poverty gap index *(PG)* counts 11.40% (see: Annex 2, Table A2.3) and shows that, on average, per month, it takes 1304,16 denars (or about 25US$) per household, for poor households to get out of the poverty zone. It means that it takes approximately 217.664.301,00 denars (or about 3.957.532,00 US$) per month, for poor households to pass the poverty line. The poverty severity index counts 8.49%.[6]

The elasticities of poverty measures with respect to households' mean income indicate that the increase of the mean income for 1% leads to decrease of the head-count index for about 1.23%, and decrease of the poverty gap index for 1.32% - which implies a high level of responsiveness of the poverty indices (see: Annex 2, Table A2.4).

The elasticities of poverty measures with respect to Gini index show that the increase of the Gini index for 1% results in increase of the head-count index for about 0.82%, and increase of the poverty gap index for 2.54% - which, as in the previous case, implies a high level of responsiveness of the poverty indices (see: Annex 2, Table A2.4).

5. CONCLUSION

The estimation of the Lorenz curve parameters, which functional form is assumed as a general quadratic, results with an inequality index that indicates moderate -, to high level of income inequality among households in Macedonia.

The poverty measures are estimated on the basis of the household's mean income (19.073,70 denars, or about 347,00 US$) and the poverty line that is set to a 60% of the household's mean income (11.444,00 denars, or about 208,00 US$). Therefore, the head-count index equals 33.38%, the poverty gap index - 11.40%, while the poverty severity index - 8.49%.

From aforementioned, a somewhat controversial conclusion for the relationship between the Macedonian poverty rate and inequality index, should be withdrawn. Namely, the high level of poverty, accompanied with the moderate level of inequality and low households' mean income, suggests that the only "thing" that should be treated as a relatively equally distributed among the households in Macedonia, actually is the poverty.

The elaticitices of poverty measures show high level of responsiveness of the poverty indices in respect with the mean income of households. The same conclusion is valid for the elasticitices of poverty measures in respect with the Gini index. We believe that these findings might be of a particular interest to the creators of economic and social policy in the Republic of Macedonia, since they lead to the conclusion that the transmission mechanism, from the economic growth towards the poverty reduction - works properly. Furthermore, they confirm that, in the case of Macedonia, does not exist, so-called, ruthless growth.

A further analysis of poverty in Macedonia requires decomposition of the changes in poverty rate into a growth and redistribution component [17], [18].

NOTES

[3] If the poverty line and mean income change in same proportion, poverty will remain unchanged.
[4] Higher value of the parameter α means higher sensitivity of the measure with respect to the inequality of the poor households.
[5] FGT belongs to the class of additively separable poverty indices, which means that deprivation that one household feels depends only on a fixed poverty line and its level of welfare, but not on the welfare of other households. So, if z is the poverty line, n is the number of households, x_i is the level of welfare of the household i, and $\psi(z,x_i)$ is the indicator of deprivation at the household's level, than this class poverty measures give the average deprivation of the total number of households: $P(z,x)=\frac{1}{n}\sum_{i=1}^{n}\psi(z,x_i)$. When the households are divided into groups, this class of measures allows one to compute the overall poverty as a weighted average of poverty in each group. The weights here are equal to households' shares. Thus, such indices are also additively decomposable [5].
[6] This index is useful for intertemporal comparison of the severity of poverty in certain country, or for country ranking.

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ANNEX 1.

	GQ Lorenz curve
Lorenz curve equation $L(p)$	$L(1-L)=a(p^2-L)+bL(p-1)+c(p-L)$ or $L(p)=-\frac{1}{2}\left[bp+e+(mp^2+np+e^2)^{1/2}\right]$
(H)	$H=-\frac{1}{2m}\left[n+r(b+\frac{2z}{\mu}\{(b+2z/\mu)^2-m\}^{-\frac{1}{2}}\right]$
(PG)	$PG=H-(\mu/z)L(H)$
(PS)	$PS=2(PG)-H-\left(\frac{\mu}{z}\right)^2\left[aH+bL(H)-\left(\frac{r}{16}\right)\ln\left(\frac{1-H/s_1}{1-H/s_2}\right)\right]$
Note:	$e=-(a+b+c+1)$ $m=b^2-4a$ $n=2be-4c$ $r=(n^2-4me^2)^{1/2}$ $s_1=(r-n)/(2m)$ $s_2=-(r+n)/(2m)$

Table A1.1: Poverty measures derived from the parameterized *GQ* Lorenz curve
Source: Datt, G.(1998).

A theoretically valid Lorenz curve satisfies following four conditions:

1) $L(0;\pi)=0$;   2) $L(1;\pi)=1$;   3) $L'(0^+;\pi)\geq 0$;   4) $L''(p;\pi)\geq 0$ за $p\in(0,1)$

The first two conditions imply that 0 and 100 percent of the households account for 0 and 100 percent of the total income, respectively. The third and fourth conditions mean that Lorenz curve is monotonically increasing and convex.

The equations for the first and second derivative of the GQ Lorenz curve, as well as the conditions for the GQ Lorenz curve validity, are presented below:

$$L'(p) = -\frac{b}{2} - \frac{(2mp+n)(mp^2+np+e^2)^{-1/2}}{4}; \qquad L''(p) = \frac{r^2(mp^2+np+e^2)^{-3/2}}{8}$$

Condition	GQ Lorenz curve
first	$e < 0$
second	$a + c \geq 1$
third	$c \geq 0$
fourth	(1) $m < 0$ or (2) $0 < m < (n^2/(4e^2)), \quad n \geq 0$ or (3) $0 < m < -(n/2), \quad m < (n^2/(4e^2))$

Table A1.2: Conditions for theoretical validity of the Lorenz curve
Source: Datt, G.(1998).

The formulas for the elasticities of poverty measures with respect to the mean income and the Gini index are given in Table A1.3:

Elasticity of	with respect to	
	Mean income	Gini index
H	$-z(\mu HL''(H))$	$(1-z/\mu)/(HL''(H))$
PG	$1-H/PG$	$1+(\mu/z-1)H/PG$
SPG	$2(1-PG/PS)$	$2(1+(\mu/z-1)PG/PS)$

Table A1.3: Elasticities of poverty measures with respect to the mean income and the Gini index
Source: Datt, G.(1998).

ANNEX 2.

Monthly income per household in denars	*P*	*L*
0-3000	0.0730	0.00918
3001-6000	0.1320	0.02310
6001-9000	0.2470	0.06832
9001-12000	0.3610	0.13108
12001-15000	0.4730	0.21035
15001-18000	0.5430	0.27090
18001-21000	0.6400	0.37007
21001-24000	0.7110	0.45382
24001-27000	0.7580	0.51666
27001-30000	0.8520	0.65711
30001-45000	0.9340	0.81833
45001 and above	1.0000	1.00000

Table A2.1: Distribution of monthly incomes of households in Macedonia, 2012
note: *p* = cumulative proportion (or percentage) from total number of households;
L = cumulative proportion (or percentage) of monthly income

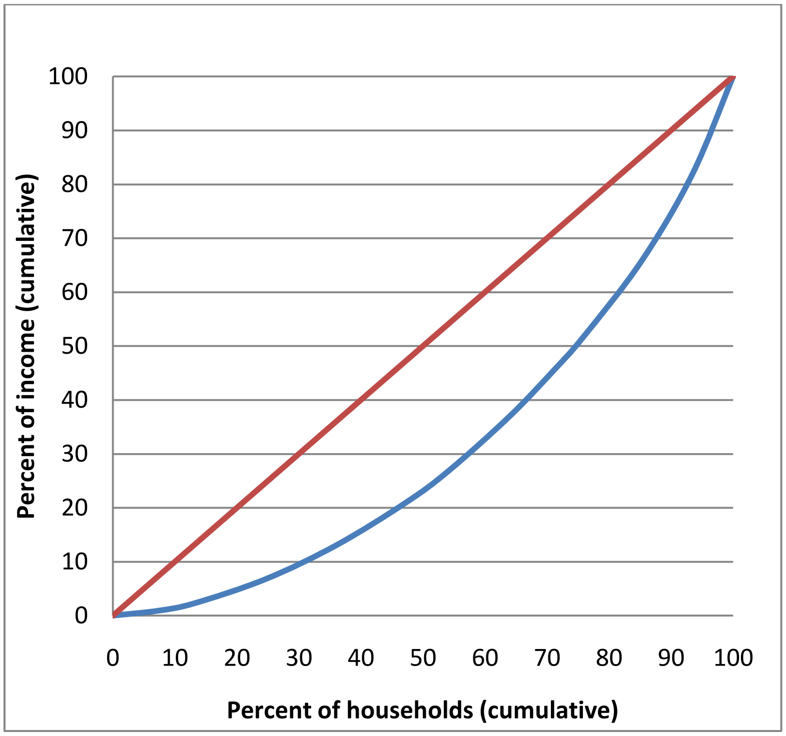

Figure A2.1: GQ Lorenz curve – Macedonia, 2012

Dependent Variable: L*(1-L)
Method: Least Squares
Sample: 1 12
Included observations: 12
L*(1-L)=a*(P^2-L)+b*(L*(P-1))+c*(P-L)

	Coefficient	Std. Error	t-Statistic	Prob.
a	1.430599	0.025493	56.11786	0.0000
b	-1.194924	0.038519	-31.02190	0.0000
c	0.062716	0.014898	4.209749	0.0023

R-squared	0.999814	Mean dependent var	0.139793
Adjusted R-squared	0.999772	S.D. dependent var	0.095691
S.E. of regression	0.001443	Akaike info criterion	-10.03128
Sum squared resid	1.88E-05	Schwarz criterion	-9.910053
Log likelihood	63.18768	Hannan-Quinn criter.	-10.07616
Durbin-Watson stat	1.754458		

Table A2.2: Regression results – *GQ* Lorenz curve

	denars
Poverty line	11444,00
Mean income	19073,70
	%
Head-count index (H)	33.38
Poverty gap index (PG)	11.40
Poverty severity index (PS)	8.49
Gini index	37.84

Table A2.3: Poverty line, mean income, poverty measures and Gini index

Poverty measures	Mean income	Gini index
Head-count index *(H)*	-1.23009	0.82010
Poverty gap index *(PG)*	-1.31852	2.54575
Poverty severity index *(PS)*	-1.39075	4.26061

Table A2.4: Elasticitices of poverty measures with respect to the mean income and the Gini index