Application of N-Period Dynamic Inventory Models with Deterministic and Probabilistic Demand

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Introduction to COMPASS

- Enterprises, driven by the global competitive environment, face the challenge of improving services, while lowering costs
- Many companies have adopted innovative business practices, to meet customer needs and retain profitability
- Enterprise restructuring is becoming very important, especially in the countries in transition
- Transition from planned to market economy is very puzzling and time consuming task
- One of the main problems is lack of appropriate knowledge for managing the enterprise in the new circumstances
- One of the most efficient ways to over-bridge this problem is to collaborate with developed countries and their enterprises and institutions, dealing with this problem
- COMPASS is a product of such research project

Introduction to COMPASS
(continued)

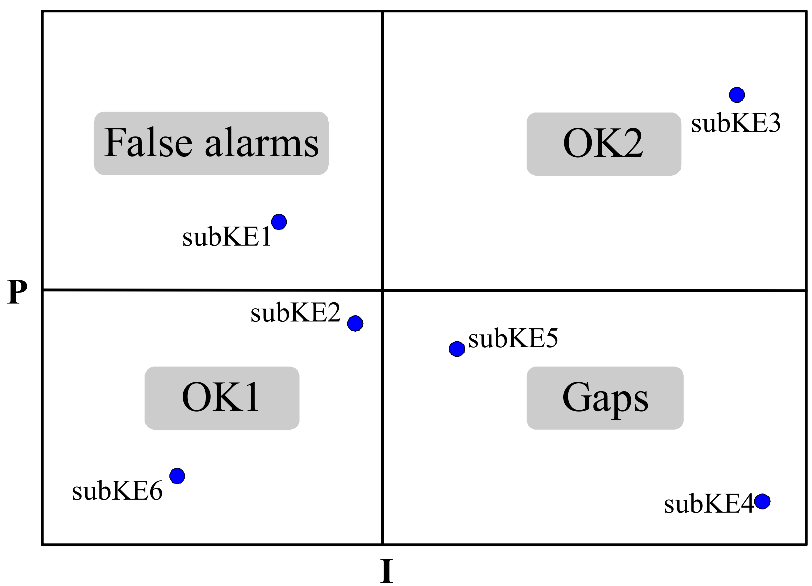
- COMPASS (COmpanyüs Management Purpose ASSistance) is a model for overall enterprise restructuring
- It is developed by the research institutions Fraunhofer Institut fuer Produktionstechnik und Automatisierung, Stuttgart, Germany, and Faculty of Mechanical Engineering, University of Ss. Cyril and Methodius, Skopje, Macedonia
- Its main intention is to offer aid to the enterprise management in direction of systematisation of the complex process of enterprise restructuring and to attach method approaches that will help in the key decision making points
- The basic idea of the model is to obtain a (sub)model of performance measurement, which will enable determination of the inconsistency of the importance and performance of all segments of the enterprise and on that basis to generate quantified alternative and than optimal actions for improvement of the situation

#	Phase in the model	Content of the phases	Some of the utilized method approaches
1.	Analysis and determination of the future market demands on the enterprise	Determination of the enterprise goals; Strategy of the enterprise; Translation of the strategy goals into KE and subKE; Ordered KE and subKE according to their strategic importance.	- AHP method - Team work (Workshop) - 7 New tools - 7 Basic tools - (sub) Model of the PMS
2.	Determination of the technical and economical capabilities	Auditing; Value of the measures; Procedure for evaluation; Ordered KE and subKE according to their actual performance.	- SAUDIT × R × measures - SWOT - Interview - Benchmarking
3.	Translation of the market demands on the enterprises	Location of the Gaps/Critical Elements × CE; List of CE.	- I/P matrices (Gap analysis) -Teamwork (Workshop)
4.	Determination of the specific capabilities and chances	Determination of the success factors; List of success factors.	- Structured knowledge about method approaches - Forms for performance measures - Matrices KE/functional areas
5.	Determination of the necessary measures (actions)	Generation of the scenarios	- Scenario technique - Qualitative MICMAC method - Simulation
6.	Quantification of the potential	Evaluation (quantification) of the scenarios; Selected scenario.	- Team work (Workshop) - Pay-back method - Costs/Gain diagram
7.	Implementation of the solution	Implementation of the scenario	

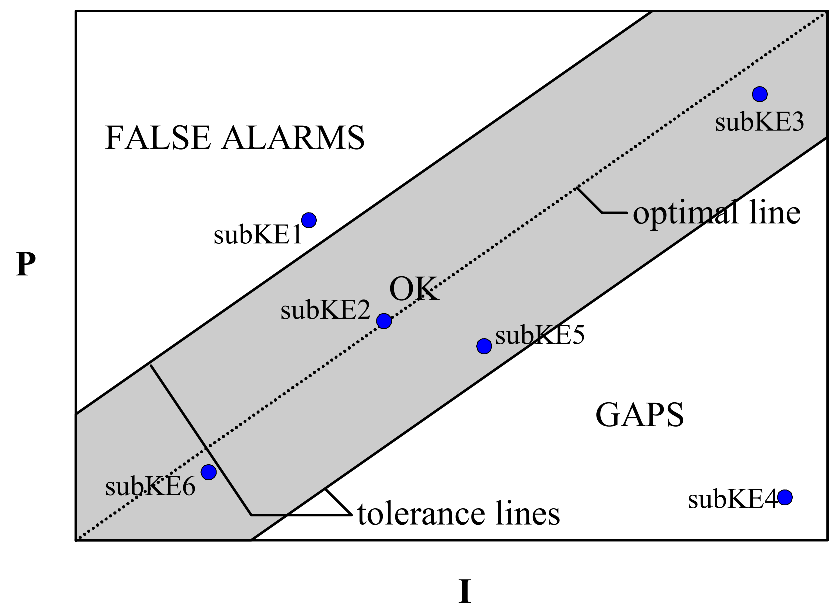
Introduction to COMPASS
(continued)

- COMPASS determines the variables-subKEs that can describe the enterprise from numerous different aspects, and measures both the strategic importance and actual performance of those variables
- SubKEs are presented in one matrix, called I/P matrix
- The output of this matrix is the list of Critical Elements (CE), subKEs which have unbalance between their importance and performance
- For every CE, appropriate Success Factor (SF) is inducted
- SFs are various kinds of actions which should lead to improved situation in the enterprise
- COMPASS tends to support the industry praxis with simple and practical methods/tools, understandable and easy to use for the majority of the employees as a way to achieve their implementation and implementation of the model as a whole
- One of its main objectives is to implement robust, scientifically funded methods which are going to help the generation of more reliable SFs, instead of heuristic generation of the SFs

I/P Matrices



False alarms
OK2
subKE3
subKE1
P
subKE2
subKE5
OK1
Gaps
subKE6
subKE4
I



FALSE ALARMS
subKE3
optimal line
subKE1
P
OK
subKE2
subKE5
GAPS
subKE6
tolerance lines
subKE4
I

Current Efforts for Improvement

- COMPASS is an open methodology, so it is giving frames of the restructuring process and it allows implementation of various appropriate methods
- Our current efforts are directed towards two main issues:
  - ✓ *The improvement of the process of generation of success factors (phase 4)*
  - ✓ *The optimization of the simulation process (phase 5)*
- At the initial development phase of COMPASS the generation of the SFs is done heuristically, because phase 4 of COMPASS is lacking concrete methods and the decisions in this phase are maid mainly heuristically
- In order to improve this phase, efforts have been made on two levels:
  - ✓ *To utilize knowledge management techniques for better structuring of the knowledge in order to induce the success factors easily*
  - ✓ *To prepare predefined solutions for some of the more frequently generated SFs, mainly using the management science methods*

Inventory Control

- Inventory is any stored resource used to satisfy a current or future need
- Most companies try to balance high and low inventory levels with cost minimization as a goal
- To control inventories means to determine HOW MUCH and WHEN to order WHICH items, so they can be used to buffer the variability and are not too expensive
- Inventory control is frequently generated as SF because of its great impact on the enterprise profitability
- The fundamental decisions in inventory control concerning the inventory policy are amenable to quantitative analysis associated with the inventory theory
- Inventory control demands parameters identification describing the dynamics of the inventory behavior over the time, using mathematical models and simulation

Inventory Models

- No unique inventory model can be developed to handle any inventory problem
- The development of numerous inventory models is implication of the most relevant factors in inventory model designing
- The type of demand (dependent or independent, deterministic or probabilistic) is principle factor
- Others are: review type, planning horizon, unsatisfied demand, relevant costs, delivery, expire date, number of items, storage capacity, lead time type, etc.
- It is rare that the available inventory models represent the inventory system accurately
- A set of most used predefined inventory models is studied and systematized in order to extend and improve COMPASS
- Besides their solution techniques, Excel spreadsheets are proposed as tools for fast and easy inventory policy determination

Inventory models (independent demand, single item)

- **Deterministic models**
  - **Continuous review (static demand)**
    - Basic EOQ model
    - EOQ model with planned shortages
    - EOQ model with quantity discounts
  - **Periodic review (dynamic demand)**
    - N-period dynamic deterministic model
- **Probabilistic models**
  - **Continuous review (stationary demand)**
    - Probabilistic continuous-review model
  - **Periodic review (nonstationary demand)**
    - Probabilistic single period models
    - N-period probabilistic models

N-Period Models

- *Continuous review case:* When the inventory level reaches the reorder point, place a new order whose size equals the order quantity
- *Periodic review case:* Receive a new order of the amount specified by the order quantity at equal time intervals
- N-period models are very close to the real life situations, and so are of great interest
- They have generated a rich mathematical theory
- In the probabilistic case they are often considered using the infinite horizon case
- Their solution techniques include dynamic programming approaches and Markov decision processes, which is very data demanding technique

N-Period Deterministic Model

- *Assumptions of the model:* known demand which varies from one period to the next, instantaneously replenishment of stock at the beginning of each period, no shortages are allowed.
- Numerical solution: dynamic programming technique
- Only finite horizon is considered
- Special cases and generalizations of this model exist
- The introduced model is not the only existing approach for the N-period deterministic model design

Inventory model

For period $i,\ i=1,2,\ldots,N$

a_i - amount ordered

d_i - amount demanded

s_i - entering inventory at the beginning of period i

h_i - holding cost per unit of inventory carried forward from period i to period $i+1$

K_i - setup cost

$c_i(a_i)$ -marginal purchasing cost function

$$C_i(a_i)=\delta_i K_i+c_i(a_i)$$

$$\delta_i=\begin{cases}0, & a_i=0\\ 1, & a_i>0\end{cases}$$

$$s_{i+1}=s_i+a_i-d_i$$

Inventory model
(Continued)

The states of the system at stage i are the amounts of entering inventory.
Backward recursive equation is:

$$f_N(s_N) = \min_{\substack{a_N + s_N = d_N \\ a_N \geq 0}} \{C_N(a_N)\}$$

$$f_i(s_i) = \min_{\substack{d_i \leq s_i + a_i \leq d_i + d_N \\ a_i \geq 0}} \{C_i(a_i) + h_i(s_i + a_i - d_i)\}, \quad i = 1, 2, \ldots, N-1.$$

The states of the system at stage i are the amounts of inventory at the end of period i.
Forward recursive equation is:

$$f_1(s_2) = \min_{0 \leq a_1 \leq d_1 + s_2} \{C_1(a_1) + h_1 s_2\}$$

$$f_i(a_{i+1}) = \min_{0 \leq a_i \leq d_i + s_{i+1}} \{C_i(a_i) + h_i s_{i+1} + f_{i-1}(s_{i+1} + a_i - d_i)\}, \quad i = 2, 3, \ldots, N.$$

$$0 \leq s_{i+1} \leq d_{i+1} + \ldots + d_N$$

N-Period Probabilistic Models

- Multiperiod (finite and infinite) probabilistic models are very connected
- They are considered under different combinations of the conditions:
  - ✓ Backlogging and no backlogging of demand
  - ✓ Zero and positive delivery lags
  - ✓ Setup and no setup costs
- *Assumptions of the model:* discounted value of money, mainly stationary demand distribution
- Numerical solution: dynamic programming formulation, optimization, Markov decision processes

Inventory model I

a_i - the inventory levels to reach by replenishing at the beginning of period i

a_i^* - optimal value of a_i

s_i - the initial inventory level (before replenishing) at the beginning of period i

$a_1^*, \ldots, a_N^*$ - optimal inventory policy, difficult to obtain numerically, but its form is:

if $s_i < a_i^*$ order $a_i^* - s_i$

if $s_i \geq a_i^*$ do not order

$$a_N^* \leq \ldots \leq a_1^* \leq a^*$$

For $N = \infty, \quad a_i^* = a^*$

$$F(a^*) = \frac{p - c(1-\alpha)}{p + h}$$ F is cumulative distribution function of the demand

Inventory model II, using MDPs approach

X_t is random variable, the state (the number) of the inventories at the end of week $t\,(t=0,1,2,\ldots)$

$D_t\,(t=1,2,\ldots)$ is random variable that represents the demand for the item and is the number of items that would be sold in week t if the inventory is not depleted, otherwise it includes lost sales

$s_{t+1}=s_t+a_t-\min\{D_t,s_t+a_t\}$ is the connection between the consecutive states

$P(D_{t+1}=j)=p_j,\ j=0,1,2,\ldots$ is the probability of demand to take a certain value

$$f_N(i)=\min_k\{C_{ik}\}$$

$$f_n(i)=\min_k\left\{C_{ik}+\sum_{j=1}^{m}p_{ij}^{k}f_{n+1}(j)\right\},\quad n=1,2,\ldots,N-1.$$

are the recursive equations

$$p_{sj}^{a}=P\{s_{t+1}=j\mid s_t=s,a_t=a\}=\begin{cases}p_{s+a-j} & j\le s+a\\ \sum_{i=s+a}^{\infty}p_i & j=0\\ 0 & j>s+a\end{cases}$$

are the transition probabilities

$$P\{D_{t+1}=j\}=\frac{(1)^j e^{-1}}{j!}\ (j=0,1,\ldots)$$

Poisson distribution with a mean of 1

$$P^0 = \|p_{ij}^0\| = \begin{bmatrix} \sum_{i=0}^{\infty} p_i & 0 & 0 & 0 \\ \sum_{i=1}^{\infty} p_i & p_0 & 0 & 0 \\ \sum_{i=2}^{\infty} p_i & p_1 & p_0 & 0 \\ \sum_{i=3}^{\infty} p_i & p_2 & p_1 & p_0 \end{bmatrix}$$

$$P^0 = \|p_{ij}^0\| = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.632 & 0.368 & 0 & 0 \\ 0.264 & 0.368 & 0.368 & 0 \\ 0.08 & 0.184 & 0.368 & 0.368 \end{bmatrix}$$

$$P^1 = \|p_{ij}^1\| = \begin{bmatrix} \sum_{i=1}^{\infty} p_i & p_0 & 0 & 0 \\ \sum_{i=2}^{\infty} p_i & p_1 & p_0 & 0 \\ \sum_{i=3}^{\infty} p_i & p_2 & p_1 & p_0 \\ - & - & - & - \end{bmatrix}$$

$$P^1 = \|p_{ij}^1\| = \begin{bmatrix} 0.632 & 0.368 & 0 & 0 \\ 0.264 & 0.368 & 0.368 & 0 \\ 0.08 & 0.184 & 0.368 & 0.368 \\ - & - & - & - \end{bmatrix}$$

$$P^2 = \|p_{ij}^2\| = \begin{bmatrix} \sum_{i=2}^{\infty} p_i & p_1 & p_0 & 0 \\ \sum_{i=3}^{\infty} p_i & p_2 & p_1 & p_0 \\ - & - & - & - \\ - & - & - & - \end{bmatrix}$$

$$P^2 = \|p_{ij}^2\| = \begin{bmatrix} 0.264 & 0.368 & 0.368 & 0 \\ 0.08 & 0.184 & 0.368 & 0.368 \\ - & - & - & - \\ - & - & - & - \end{bmatrix}$$

$$P^3 = \|p_{ij}^3\| = \begin{bmatrix} \sum_{i=3}^{\infty} p_i & p_2 & p_1 & p_0 \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix}$$

$$P^3 = \|p_{ij}^3\| = \begin{bmatrix} 0.08 & 0.184 & 0.368 & 0.368 \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix}$$

Conclusions

- The solution obtained from an inventory model should be regarded as a guideline for the managers rather than a specific recommendation for decision making
- To ensure the reliability of the recommended solution to the complex inventory problem, it is useful to model the system by simulation
- Probabilistic models are far more complex than deterministic models and closer to the real situations
- One can use the models without knowing the theory, since the calculations and the results for the optimal policy can be easily obtained using programs in Excel, MATLAB, LINDO/LINGO, CPLEX,Ñ etc.