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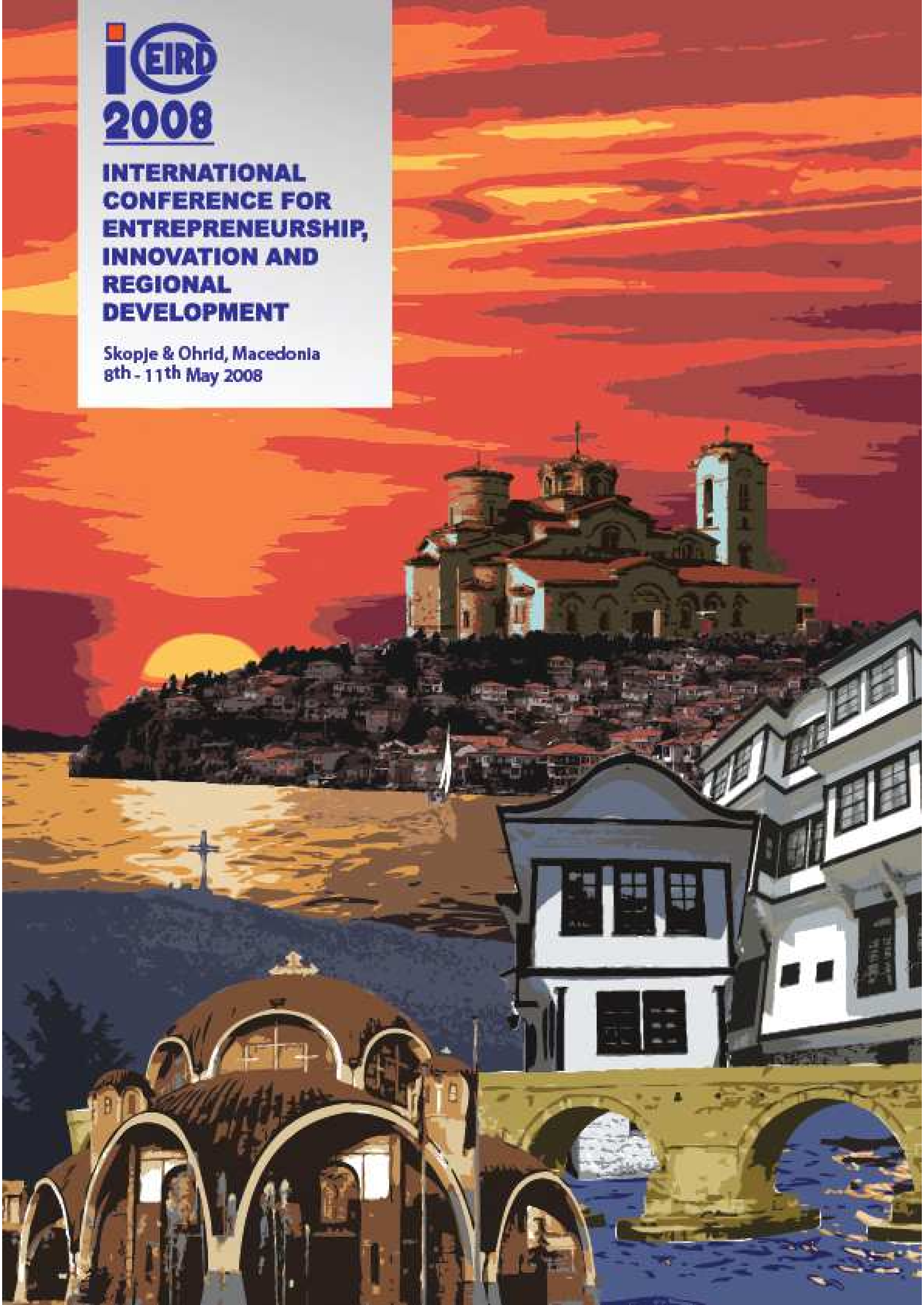
2008

INTERNATIONAL CONFERENCE FOR ENTREPRENEURSHIP, INNOVATION AND REGIONAL DEVELOPMENT

Skopje & Ohrid, Macedonia
8th - 11th May 2008



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INTERNATIONAL CONFERENCE FOR ENTREPRENEURSHIP, INNOVATION AND REGIONAL DEVELOPMENT

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8th - 11th May 2008
SKOPJE & OHRID, MACEDONIA

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Proceedings of the International Conference for
Entrepreneurship, Innovation and Regional Development
ICEIRD 2008

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Cover Design	4ward

Publisher:
Business Start-up Centre, University "Ss. Cyril and Methodius" – Skopje

200 copies

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WELCOME MESSAGE

Dr. Radmil M. POLENAKOVIK
ICEIRD2008 – President

Dr. Panos H. KETIKIDIS
ICEIRD2008 – Vice President

Dear Friends, Colleagues

It is a great pleasure for us, the University “Ss. Cyril and Methodius” Business Start-up Centre in Skopje, together with CITY College - Computer Science Department, Thessaloniki to host **First International Conference for Entrepreneurship, Innovation, and Regional Development (ICEIRD-2008).**

The Conference contributes directly to the mission of ICEIRD which is to support the development of South East Europe by providing a multi-disciplinary forum for researchers, practitioners and policy makers in the field of innovation/entrepreneurship and regional development and a means for sharing findings that promote innovation and therefore enhance economic, technological and regional development through new economic activities that stimulate generation of wealth employment and growth and increase competiveness.

ICEIRD was the result of a collective effort and we would like to take the opportunity to express my appreciation to the people involved. First of all, we would like to thank the Organising and scientific Programme Committee as well as all the authors, speakers, reviewers and participants whose contribution to the conference made it a success. We would like to thank our colleagues from the University “Ss. Cyril and Methodius” in Skopje, together with CITY College in Thessaloniki for devoting a valuable personal time to the organisation of this event and especially Mr. Kurciev, Mr. Jovanoski, Mr. Velkovski and Mr. Jovanovski from the University “Ss. Cyril and Methodius” Business Start-up Centre and Mr Thanos Hatziapostolou, Director for the MSc in Technology, Innovation and Entrepreneurship at CITY College as well as Mrs Sotiriadou, Head of Computer Science at CITY College.

Finally, we would like to warmly thank our sponsors: the Austrian Development Agency - Austria, SPARK - Nederland, ASO Offices in Sofia, Bulgaria and Ljubljana - Slovenia, Ministry for Education and Science of the Republic of Macedonia, Agency for Promotion of the Entrepreneurship in the Republic of Macedonia, GTZ REDEM office in Skopje and the South East European Research Centre (SEERC) in Thessaloniki, Greece. Without their support this event would not be possible. Also we would like to express our gratitude to our host institutions - Faculty of Mechanical Engineering and University “Ss. Cyril and Methodius” and CITY College an affiliated institution of the University of Sheffield, for all necessary support that we received during preparation of the Conference.

We are looking forward to cordially welcome you all to the 2nd ICEIRD that will take place in Thessaloniki, Greece next year.

Prof. Dr. Panos H. KETIKIDIS

Vice Principle - CITY College
(an affiliated institution of
the University of Sheffield)

Prof. Dr. Radmil Polenakovik

Director - Ss. Cyril and
Methodius University Business
Start-Up Centre

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(in alphabetical order by author last name)

INTERNATIONAL CONFERENCE FOR ENTREPRENEURSHIP, INNOVATION AND REGIONAL DEELOPMENT - ICEIRD 2008

The University "Ss. Cyril and Methodius" Business Start-up Centre in Skopje, Macedonia together with CITY College (an affiliated institution of University of Sheffield) - Computer Science Department, Thessaloniki, Greece are organizing the first International Conference for Entrepreneurship, Innovation, and Regional Development ICEIRD 2008. It is taking place in Skopje and Ohrid, MACEDONIA in the period 8th – 11th May 2008.

ICEIRD 2008 is organized in conjunction with the EU Day of the Entrepreneur - 9th May, 4th SENSI (South-East European Network of Start-ups and Incubators) meeting (www.sensi.biz) and 133rd SPICE (Science Park and Innovation Center Experts) meeting (www.spice-group.de).

Topics that are present at the Conference covered:

- Governmental and regional policies on entrepreneurship, innovation and R&D
- Clustering and networking
- Entrepreneurship education
- Family businesses and entrepreneurship
- University – industry collaboration
- Innovation policy in small and medium enterprises
- ICT and Regional competitiveness
- Knowledge management and technology transfer
- Regional innovation strategies
- Regional competitiveness and development
- Business process modelling
- The benefit of knowledge zone, business centres and incubators in the region
- Business incubation
- Best practices in the region / Macedonia (start-up companies)
- Financing innovation, R&D, SMEs

Objective of the conference

The objective of the Conference is to gather in the same place decision makers (government, ministries and state agencies), scientists (universities, research and development centres, start-up centres and incubators) and practitioners (SME's) in order to discuss topic that are of crucial importance for national competitiveness and increased regional development in the South East Europe.

The key areas of the conference are:

- **Entrepreneurship** as a process of identifying opportunities and putting useful ideas into practice;
- **Innovation** as the driver of national, regional and global economy;
- **Regional development** and the possibilities and barriers for closer cooperation between South East European economies.

Target Audience

The conference is addressed at national and regional government representatives in all countries of South-East Europe, who are involved in the process of policy making in the area of Innovation, Entrepreneurship and Regional Development. Special target group are enterprises, as well as non-governmental organizations active in conference topics.

The conference brings together policy makers, experts, practitioners, professors, business people and scientists in this subject area. ICEIRD 2008 will make a contribution to policy making and new ideas on competitiveness in the region. Special target audience are students, young researchers, scientists, and their supervisors from academia and industry to present actual research projects and results.

NEXT CONFERENCES

ICEIRD 2009 – Thessaloniki, Greece (organized by CITY College, an affiliated institution of University of Sheffield)

ICEIRD 2010 – Novi Sad, Serbia (organized by Faculty of Technical Sciences, University of Novi Sad, Serbia)

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Finite-stage Markov Decision Processes in Inventory Management

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Markov decision processes are a flexible technique for stochastic and dynamic optimization problems and provide a powerful tool for optimizing their performance. Their applications arise in inventory management, cash-flow management, water resources management, reliability and replacement problems, some routing and logistics problems, health care, highway and bridge maintenance, machine maintenance, forest management, control of queuing systems, operation of communication networks,… etc. They are very pervasive and many companies use them. The increased dynamics of the market is making the problem of valid and fast decision making more and more actual. So is the problem of the determination of the enterprise performance. Ability of the enterprise to continuously adapt to the market changes is inevitable for the enterprise success. The main intention of our model for enterprise restructuring, named as COMPASS is to systematise the complex process of enterprise restructuring and to attach appropriate methods in the key decision making points, supporting the industry praxis with simple and practical methods/tools, which will be understandable and easy to use, such as Markov decision processes model. That is the only way to achieve implementation of the methods and the model as a whole. Here we introduce an application of finite-stage Markov decision processes in an inventory problem. The example is an equivalent to a number of important applications and the idea of the example can be adapted to represent this applications. Besides the mathematical background, one can use the model without knowing the theory, since the calculations and the results for the optimal policy and the value function can be obtained easily using programs in Excel, MATLAB, LINDO/LINGO, CPLEX,…etc.

Keywords

Inventory management, Markov decision processes, Policy, Transition probabilities, Value iterations.

1. Introduction

Markov decision processes are a method for formulating and solving stochastic and dynamic decisions. They have a long history of applications in inventory management, cash-flow management, water resources management, reliability and replacement problems, some routing and logistics problems, health care, highway and bridge maintenance, machine maintenance, forest management, control of queuing systems, operation of communication networks, … etc, and many companies use them in some form or another. Their advantage in modelling is their flexibility, which may allow them to be useful for certain problems, but from a solution viewpoint it is their disadvantage, because they can't take advantage of special structure. It is unlikely that they can work as stand alone techniques; which enables us to do some comparisons between some management science methods. The implementation of these techniques in our model for overall restructuring of the enterprises, COMPASS, as extension to the model, is one of the main objectives.

The picture about the actual situation in the enterprise is described through the variables of the COMPASS-subKEs. SubKEs are presented in one matrix, called I/P (Importance/Performance).The output of this matrix is the list of Critical Elements-subKEs which have unbalance between their importance and performance. I/P matrices are genuinely gap analysis, presented in portfolio way, which improves the transparency of the analysis. For every Critical Element (CE) appropriate Success Factor (SF) is inducted. SFs are various kinds of actions which should lead to improved situation in the enterprise. At the initial development phase of COMPASS the generation of the SFs is done heuristically [6]. As we previously mentioned, the idea is to improve COMPASS with scientifically funded methods which are going to help the generation of more reliable SFs. In that direction, Markov decision processes are used here to support the inventory policy determination. These article intents to give an introduction to their application in inventory management, where they have a particular success. Markov decision processes are at the heart of every inventory management problem. Here we use a finite-stage Markov decision processes model in order to illustrate its application on an inventory example. The objective is to find the optimal inventory policy and to determine the utility function. There is no need for mathematical theory background for using this algorithm. It is easily applied using programs in Excel, MATHLAB, LINDO/LINGO, CPLEX, etc.

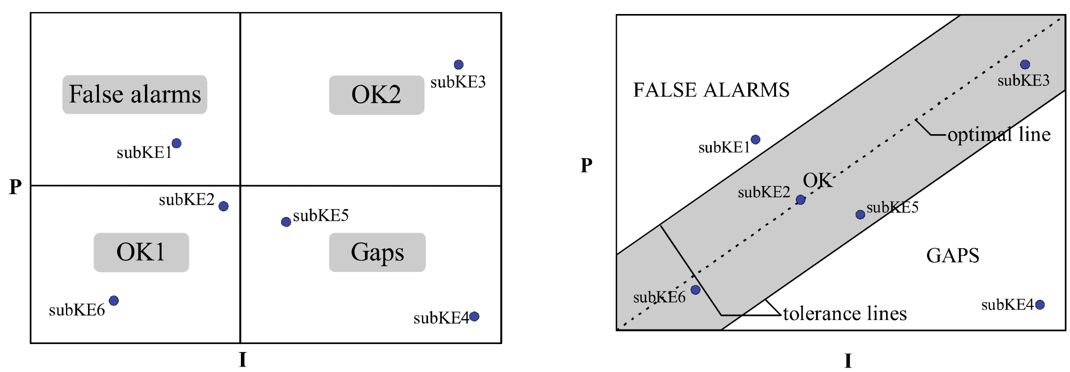

Figure 1 Two alternative ways of presenting the Importance/Performance (I/P) matrixes

2. Characteristics of the Markov decision processes

Markov decision process model has generated a rich mathematical theory. Sometimes this model may appear quite simple and it encompasses a wide range of applications. We will give an introduction of the Markov decision processes vocabulary in this section. Decision epochs (t) are points in time at which decisions are made. States are relevant information needed to describe the system. The set of possible states is the state space (S). Actions are means by which the decision maker interacts with the system. When the system is in state i (or we can use s_t as notation), the decision maker chooses an action (decision) (a) from a certain action set ($A(i)$), which may depend on the observed state. Given the state of the system and the chosen action, the immediate reward (or cost) (r (or c)) is earned, and it doesn't depend on the history of the process. The action chosen affects both the immediate rewards (costs) and subsequent rewards(or costs). In different sources, we can find Markov decision processes models including rewards or costs, because there is not an essential difference between rewards and costs, since maximizing rewards is equivalent to minimizing costs, except for the fact that costs data are easier to find. Given the state of the system (i)

and the chosen action, the state at the next decision time point (j) is determined by a transition law given by the transition probability (p_{ij}). Transition probabilities are distribution that governs how the state of the process changes as actions are taken over time and they posses the Markov property. The planning horizon of the process may be finite, infinite or of random length. The decision rule is a rule for a particular state that prescribes an action for each decision epoch. The value function (utility function) helps to determine maximum total expected reward (or minimum total expected cost).

Collectively, decision time points (epochs) (t), states, actions, rewards (costs) and transition probabilities form a Markov decision process. *Policy* is a collection of decision rules for all states. Under a fixed policy, the process behaves according to a Markov chain.

2.1 Markov decision processes solution techniques

There are various factors to consider in choosing the right technique for solving a Markov decision process problem. A division is made according to whether the problem is formulated as a finite or infinite horizon (stage, period) problem.

If the problem is formulated as a finite horizon problem, the choice of the solution technique does not depend on discounting of the rewards (costs), or whether the objective is to maximize (minimize) expected total reward (cost) or to maximize (minimize) expected average reward (cost). Here the solution technique is backwards induction solution technique, which uses recursive equations of the dynamic programming. The same solution technique can be presented with directed graph, and the problem of finding the shortest path (and its length) can be solved as a Markov decision process over a finite horizon. This solution technique Is also referred to as value iteration approach, or method of successive approximations. This is approach for quickly finding at least an approximation to an optimal policy.

But for infinite horizon, technique does depend on discounting and the objective. Here can be considered exhaustive policy enumeration, value iteration, policy iteration (policy improvement algorithm) and linear programming approach.

3. Finite horizon problems

We consider a problem solved directly with dynamic programming recursive equations, a model developed for solving a finite horizon (finite-stage) Markov decision problem. In the finite horizon case one has to control a system with, in general, non-stationary rewards (costs) and non-stationary transition probabilities, over a finite planning horizon of N periods. We consider the total expected reward (or cost) as utility (value) function. An optimal Markov policy, with deterministic, but in general non-stationary decision rules exists, and such an optimal policy can be obtained by backward induction, which is based on the principle of optimality and it is an iterative approach starting at the end of the planning horizon. Here we will give the idea behind backwards induction and the formalization of the same illustrated on an inventory example, in order to find a policy that maximizes (minimizes) a certain value function. The idea is first to observe the last time period for all the possible states and decide the best action for all those states, which enables us to gain an optimal value for that state in that period. Next, we go in the next-to-last period for all the possible states and decide the best action for those states, knowing and using the optimal values of being in various states at the next time period. And we continue this process until we reach the present time period.

3.1 Inventory problem statement and its Markov decision processes model

Most managers don't like inventories since they are tied up in investments that are not producing any return, but incurring a borrowing cost, costs for the care of the stored material and are subject to spoilage and obsolescence. There are many programs developed by industry, aimed at reduction inventory levels and increasing efficiency on the shop floor, and we only mention the most popular, conwip, just-in-time manufacturing, lean manufacturing, flexible manufacturing, …etc. But, in spite of the bad features, inventories do have positive purposes such as providing a stable source of input required for production, reduction of ordering costs, reduction of the impact of the variability of the production rates in a plant, protection against failures in the processes, better customer service, variety and easy availability of the product, …etc. Inventories have practical and economic importance and significant portion of almost any company's assets comprises inventories. The subject of inventory control is a consideration in many situations and the questions for the inventory policy are amenable to quantitative analysis associated with the inventory theory. Here we observe a management science method for inventory policy determination, an application of Markov decision processes, very close to real inventory situations and applicable in practice.

First we state a very common inventory problem, with its characteristics and simplifying assumptions, that is going to be solved as a discrete time Markov decision process problem and discuss why it possesses the Markov property. We give the general model for solving the problem, and only mention a generalized version of the problem that can be developed. In Markov models we are often interested in cost calculations. Particularly, in an inventory model they are the storage costs.

Each week, the inventory manager of a store observes current inventory on hand of a single product, and decides how much additional stock to order from a supplier. Although the weekly demand is uncertain, we know the probability distribution.

As we know, in practice, it is easier to minimize the costs for the inventories, since they are easier to determine in advance, than to maximize the rewards, for they are difficult to evaluate. There are many costs considered in inventory theory, but in this model we consider only the relevant ones, inventory holding and lost sales, for there is no backlogging, and the excess demand is lost. The mentioned generalized problem removes this assumption and it is inventory model with backlogging and we only mentioned it as an option.

The store is in a position for instantaneous delivery, and that solves the problem with the limited capacity of M units. There are no changes in revenues, costs and demand from week to week.

The random variable X_t is the state (the number) of the inventories at the end of week $t\left(t = 0,1,2,\ldots\right)$. X_0 represents the number of items on hand at the outset. So, the state at time t equals the number of items at the end of week t. The random variables X_t are dependent.

The random variable $D_t\left(t = 1,2,\ldots\right)$ represents the demand for the item and is the number of items that would be sold in week t if the inventory is not depleted, otherwise it includes lost sales. For this model's relevance we assume that D_t are independent and identically distributed random variables. Given that the current state is $X_t = i$, X_{t+1} depends only on D_{t+1}, and since it is independent of any past history of the system prior to time t, the stochastic process $\{X_t\}\left(t = 0,1,\ldots\right)$ has the Markovian property, and so is a Markov chain.

We give the essential notations and equations for our Markov decision process model:

The connection between the consecutive states is $s_{t+1} = s_t + a_t - \min\{D_t, s_t + a_t\}$;

The probability of demand to take a certain value is $P\left(D_{t+1} = j\right) = p_j$, $j = 0,1,2,\ldots$;

If we go with the model with maximizing the expected rewards, then a reward equals expected income, minus relevant costs (order cost, holding cost, lost sales cost, respectively):

$$r_{s_t,s_{t+1}} = r_t(s_t,a_t) = \sum_{j=0}^{s_t+a_t} g(j)p_j + g(s_t+a_t)\sum_{j=s_t+a_t}^{\infty} p_j - O(a_t) - h(s_t+a_t) - l(D_t),$$

where g symbolizes the function for sales income.

$$l(D_t) = \begin{cases} D_t - (s_t+a_t), & D_t > s_t + a_t \\ 0, & D_t \le s_t + a_t \end{cases}$$

The recursive equations are given with the formulas:

$$v_i^k = \sum_{j=1}^{m} p_{ij}^k r_{ij}^k,$$

where

$$f_N(i) = \max_k \{v_i^k\}$$

$$f_n(i) = \max_k \left\{ v_i^k + \sum_{j=1}^{m} p_{ij}^k f_{n+1}(j) \right\}, \quad n = 1, 2, \ldots, N-1.$$

But if we go minimizing the relevant costs, we can use the following recursive equations:

$$f_N(i) = \min_k \{C_{ik}\}$$

$$f_n(i) = \min_k \left\{ C_{ik} + \sum_{j=1}^{m} p_{ij}^k f_{n+1}(j) \right\}, \quad n = 1, 2, \ldots, N-1.$$

to solve our example, just to illustrate costs determination and the iterations. One can see the similarity of the proposed iterations.

The transition probabilities are given with $p_{sj}^a = P\{s_{t+1} = j \mid s_t = s, a_t = a\} = \begin{cases} p_{s+a-j} & j \le s+a \\ \sum_{i=s+a}^{\infty} p_i & j = 0 \\ 0 & j > s+a \end{cases}$

In some Markov decision processes problems, a thorough inspection is done at every decision epoch that results in classifying the condition into one of the possible states. After historical data on these inspection results are gathered, statistical analysis is done on how the state of the system evolves from a decision epoch to the next. This is how the relative frequency (probability) of each possible transition from the state in one epoch to the state in the following epoch is gained. These transition probabilities form the transition probability matrix. But as we mentioned before, the transition probabilities in our inventory example depend on demand probability distribution and are determined using probability theory, so the polices are randomized.

3.2 Inventory problem solution

The reason that this approach is attractive is that there is a quick method of finding an optimal policy when the process has only N periods to go, and that is the probabilistic dynamic programming.

In this part we give the data for a concrete example for the inventory model given previously, in order to illustrate the solution procedure. Some of them are: the inventory level fluctuates between a minimum of 0 items and a maximum of 3 items, so the possible states of the system at time t (the end of week t) is the state space $S = \{0,1,2,3\}$, i. e. the only possible values for the random variable X_t are 0, 1, 2 or 3.

The action sets are: $A(0) = \{0,1,2,3\}$, $A(1) = \{0,1,2\}$, $A(2) = \{0,1\}$, $A(3) = \{0\}$, since we have a limited space of $M = 3$ units, so $s_t + a_t \le M \quad (t = 0,1,2,\ldots)$.

Table 1 Classifying the condition of the system in four states

State	Condition
0	0 items on hand
1	1 item on hand
2	2 items on hand
3	3 items on hand

Table 2 Classifying the actions depending on states

Decision	Action	Relevant states
0	0 items to order	0,1,2,3
1	1 item to order	0,1,2
2	2 items to order	0,1
3	3 items to order	0

An important modelling decision concerns which distribution to use for demand. Transition probabilities depend on probability distribution of demand, which for models with small state space or when the expected demand in a time interval is small is recommended to be Poisson or exponential [4]. Here we take Poisson distribution with a mean of 1:

$$P\{D_{t+1} = j\} = \frac{(1)^j e^{-1}}{j!} \; (j = 0,1,\ldots).$$

For our purposes we compute:

$$p_0 = P\{D_{t+1} = 0\} = 0.368,\; p_1 = P\{D_{t+1} = 1\} = 0.368,\; p_2 = P\{D_{t+1} = 2\} = 0.184,$$
$$P\{D_{t+1} \ge 3\} = 0.080.$$

The transition probability data according to model formulation given above can be calculated as shown with the given “matrices”, and they are not given in a sense of a transition probability matrices, but only a data matrices for the allowed transitions, and we put “–“ in the place where the transition is not allowed. Transition matrices can be obtained for a certain policy with determined and fixed decision rules, which is not the case in our representation used only for transparency. Only the matrix P^0 can be observed as transition matrix for the policy: do not make orders no matter the state, and as we’ll see later, state 0 is absorbing state.

$$P^0 = \left\| p_{ij}^0 \right\| = \begin{bmatrix} \sum_{i=0}^{\infty} p_i & 0 & 0 & 0 \\ \sum_{i=1}^{\infty} p_i & p_0 & 0 & 0 \\ \sum_{i=2}^{\infty} p_i & p_1 & p_0 & 0 \\ \sum_{i=3}^{\infty} p_i & p_2 & p_1 & p_0 \end{bmatrix},\; P^1 = \left\| p_{ij}^1 \right\| = \begin{bmatrix} \sum_{i=1}^{\infty} p_i & p_0 & 0 & 0 \\ \sum_{i=2}^{\infty} p_i & p_1 & p_0 & 0 \\ \sum_{i=3}^{\infty} p_i & p_2 & p_1 & p_0 \\ - & - & - & - \end{bmatrix},$$

$$P^2 = \left\| p_{ij}^2 \right\| = \begin{bmatrix} \sum_{i=2}^{\infty} p_i & p_1 & p_0 & 0 \\ \sum_{i=3}^{\infty} p_i & p_2 & p_1 & p_0 \\ - & - & - & - \\ - & - & - & - \end{bmatrix}, \; P^3 = \left\| p_{ij}^3 \right\| = \begin{bmatrix} \sum_{i=3}^{\infty} p_i & p_2 & p_1 & p_0 \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix}.$$

If we calculate the values of the transition probabilities, we obtain

$$P^0 = \left\| p_{ij}^0 \right\| = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.632 & 0.368 & 0 & 0 \\ 0.264 & 0.368 & 0.368 & 0 \\ 0.08 & 0.184 & 0.368 & 0.368 \end{bmatrix}, \; P^1 = \left\| p_{ij}^1 \right\| = \begin{bmatrix} 0.632 & 0.368 & 0 & 0 \\ 0.264 & 0.368 & 0.368 & 0 \\ 0.08 & 0.184 & 0.368 & 0.368 \\ - & - & - & - \end{bmatrix},$$

$$P^2 = \left\| p_{ij}^2 \right\| = \begin{bmatrix} 0.264 & 0.368 & 0.368 & 0 \\ 0.08 & 0.184 & 0.368 & 0.368 \\ - & - & - & - \\ - & - & - & - \end{bmatrix}, \; P^3 = \left\| p_{ij}^3 \right\| = \begin{bmatrix} 0.08 & 0.184 & 0.368 & 0.368 \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix}.$$

Statistical analysis can prove that these transition probabilities are unaffected by also considering what the states were in prior weeks, which is the “lack of memory” property called Markovian property.

For every transition probability a cost data should be computed according to the model formulas and initial values. The holding cost $h(s_t + a_t)$, the lost sales cost $l(D_t)$, and the order cost $O(a_t)$ depend on the variables in brackets. Purchasing cost could also be included in total costs. A discussion about the costs is made in inventory theory. Because every inventory case has its own characteristics, we shall not discuss them here, but only give arbitrary but yet sophisticated values in order to illustrate the algorithm.

Table 3 Expected total cost data for a week, caused by a certain decision

Decision (k)	**State** (i)	**Holding cost, $**	**Lost sales cost, $**	**Order cost, $**	**Purchasing cost, $**	**Total cost for a week, $,** (C_{ik})
0	0	0	6320	0	0	6320
	1	10	2640	0	0	2650
	2	20	800	0	0	820
	3	30	0	0	0	30
1	0	10	2640	10	1000	3660
	1	20	800	10	1000	1830
	2	30	0	10	1000	1040
2	0	20	800	20	2000	2840
	1	30	0	20	2000	2050
3	0	30	0	30	3000	3060

For the algorithm transparency we use the following tables containing the values from the recurrence equations given in the previous section. Let $N = 4$ (weeks), and we have a reason for choosing bigger value to approximate the optimal policy.

Table 4 Stage 4

State (i)	C_{ik}				Optimal solution	
	$k=0$	$k=1$	$k=2$	$k=3$	$f_4(i)$	k^*
0	6320	3660	2840	3060	2840	2
1	2650	1830	2050	-	1830	1
2	820	1040	-	-	820	1
3	30	-	-	-	30	0

The first approximation calls for ordering 2 items if the inventory level is 0, ordering 1 item if the inventory level is 1 or 2, and not to put an order if the inventory level is 3, and that is the optimal policy for this stage.

Table 5 Stage 3

State (i)	$C_{ik} + p_{i0}^k f_4(0) + p_{i1}^k f_4(1) + p_{i2}^k f_4(2) + p_{i3}^k f_4(3)$				Optimal solution	
	$k=0$	$k=1$	$k=2$	$k=3$	$f_3(i)$	k^*
0	9160	7285.248	4564.96	3936.72	3936.72	3
1	5118.32	3554.96	2926.72	-	2926.72	2
2	2544.96	1916.72	-	-	1916.72	1
3	906.72	-	-	-	906.72	0

The second approximation calls for ordering 3 items if the level is 0, 2 items if the level is 1, 1 item if the level is 2, and 0 items if the inventory level is 3. That is the optimal policy for this stage. This policy recommendation continues in the next two stages and that is probably the optimal policy for this example for the infinite horizon case, but we can not prove this unless we use other approaches. $f_n(i)$ is the expected total cost from the stages $n, n+1, ..., N$, if the process starts at state i at the beginning of the week n.

Table 6 Stage 2

State (i)	$C_{ik} + p_{i0}^k f_3(0) + p_{i1}^k f_3(1) + p_{i2}^k f_3(2) + p_{i3}^k f_3(3)$				Optimal solution	
	$k=0$	$k=1$	$k=2$	$k=3$	$f_2(i)$	k^*
0	10256.72	7225.04	5661.68	4952.48	4952.48	3
1	6215.04	4651.68	3942.48	-	3942.48	2
2	3641.68	2932.48	-	-	2932.48	1
3	1922.48	-	-	-	1922.48	0

Table 7 Stage 1

State (i)	$C_{ik} + p_{i0}^k f_2(0) + p_{i1}^k f_2(1) + p_{i2}^k f_2(2) + p_{i3}^k f_2(3)$				Optimal solution	
	$k=0$	$k=1$	$k=2$	$k=3$	$f_1(i)$	k^*
0	11272.48	8240.8	6677.44	5968.24	5968.24	3
1	7230.8	5667.44	4958.24	-	4958.24	2
2	4657.44	3948.24	-	-	3948.24	1
3	2938.24	-	-	-	2938.24	0

Total expected costs for the four observed weeks are $f_1(0) = 5968.24$, if the state at the beginning of the week 1 is 0, $f_1(1) = 4958.24$ if the state is 1, …etc.

We define stationary policy and discuss policy enumeration, as a method for problem analysis and for best policy choice. The decision making process evaluating the expected revenue (lost) resulting from a prespecified course of action for a given state of the system is said to be represented by a stationary policy. Evaluation of all possible stationary policies of the decision problem which is equivalent to an exhaustive enumeration process, can be used only if the number of stationary policies is reasonably small and mostly for the infinite horizon problems. We emphasize that this method can be impractical, even for the limited size problems.

4. Conclusions

As N grows large, the corresponding optimal policies will converge to an optimal policy for the infinite-period problem. Although the method of successive approximations may not lead to an optimal policy for the infinite-stage problem after a few iterations, it never requires solving a system of equations. This is its advantage over the policy improvement and linear programming solution techniques, for its iterations can be performed simply and quickly. But it definitely obtains an optimal policy for an n-period problem after n iterations [7].

As the problem size increases, i. e. the state and/or the action space become larger, it becomes computationally very difficult to solve the Markov decision processes problem. There are some methods that are more memory-efficient than policy iteration and value iteration algorithms. There are some solution techniques that find near-optimal solutions in short time. For each action and state pair, we need a transition probability matrix and a reward function, which are enormous data requirements. Infinite-stage Markov decision process problems can be formulated and solved as linear programs. Also we can find approximate solution techniques that are promising. There are extensions of Markov decision processes, because of their limitations. But the finite-stage Markov decision processes problems are more likely to be found in reality of inventory management, where there is a recursive nature to the problem. In practice it is not, usual to have infinite planning horizon. That is the reason why we set out this model which is very close to real situations in inventory management, but yet easy to analyze and understand. And as we mentioned, there is no need to know the mathematical theory for its implementation, because there is the opportunity for applicable computer programs.

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