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For publisher:
Milan Pavlovic, Ph. D, Professor, Dean of the Technical faculty „Mihajlo Pupin”, Zrenjanin

Technical preparation and design:
Ivan Tasic, Ph. D, Assistant Professor
Dijana Karuovic, Ph. D, Assistant Professor
Marjana Pardanjac, Ph. D, Assistant Professor
Erika Eleven, M.Sc, Assistant
Dusanka Milanov

Lecturer:
Erika Tobolka, Ph. D, Professor

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УНИВЕРЗИТЕТ
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With this publication, the CD with all papers from the International Conference on Information Technology and Development of Education, ITRO 2014 is also published.

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INTRODUCTION

This Proceedings comprises papers from the **International conference on Information technology and development of education** that is held in the National House of Mihajlo Pupin, Idvor on June 27th 2014.

The International conference on Information technology and development of education has had a goal to contribute to the development of education in Serbia and in the region, as well as, to gather experts in natural and technical sciences' teaching fields.

The expected scientific-skilled analysis of the accomplishment in the field of the contemporary information and communication technologies, as well as analysis of state, needs and tendencies in education all around the world and in our country have been realized.

The authors and the participants of the Conference have dealt with the following thematic areas:

- Theoretical and methodological questions of contemporary pedagogy
- Personalization and learning styles
- Social networks and their influence on education
- Children security and safety on the Internet
- Curriculum of contemporary teaching
- Methodical questions of natural and technical sciences subject teaching
- Lifelong learning and teachers' professional training
- E-learning
- Education management
- Development and influence of IT on teaching
- Information communication infrastructure in teaching process

All submitted papers have been reviewed by at least two independent members of the Science Committee.

The papers presented on the Conference and published in this Proceedings can be useful for teacher while learning and teaching in the fields of informatics, techniques and other teaching subjects and activities. Contribution to science and teaching development in this region and wider has been achieved in this way.

The Organizing Committee of the Conference

CONTENTS

APPLYING THE FUNDAMENTAL LEMMA OF VARIATIONAL CALCULUS TO THE PROBLEM OF THE SMALLEST SURFACES IN ROTATION

A. Risteska, V. Gicev
Faculty of Computer Science, University Goce Delcev, Stip, Republic of Macedonia

Abstract - Variational calculus studies methods for finding maximum and minimum values of functional. It has its inception in 1696 by work of Johan Bernoulli with its glorious problem for the brachistochrone: to find a curve, connecting two points A and B , which does not lie in a vertical, so that heavy point descending on this curve from position A to reach position in B for the least time. In functional analysis variational calculus takes the same space, as well as theory of maximum and minimum intensity in the classic analysis. We use and prove the theoremfor functional where we prove that necessary condition for extreme of functional is the variation of functional to be equal to zero.We have simplified the received necessary condition for extreme and prove so-called, the main lemma of variational calculus. At least we describe private case in the solution of the equation of Euler and give an example of application, such asthe problem of the smallest surfaces in rotation.

I. THEORETICAL ISSUES

We are searching for extreme of the functional

$$v[y(x)] = \int_{x_0}^{x_1} F(x, y(x), y'(x))\, dx, \qquad (0.1)$$

with the limit points of the allowable set of curves: $y(x_0) = y_0$ and $y(x_1) = y_1$. We will consider that the function $F(x, y, y')$ isthree times differentiable.It is known that necessary condition for extreme of the functional (0.1) is that its variation is equal to zero.We will now show how the main theorem is applied to the given functional (0.1). We are taking some limit curves $y = \overline{y}(x)$ close to $y = y(x)$ and include curves $y = y(x)$ and $y = \overline{y}(x)$ to the family curves with one parameter

$$y(x,\alpha) = y(x) + \alpha(\overline{y}(x) - y(x)).$$

When $\alpha = 0$ we receive the curve $y = y(x)$, when $\alpha = 1$ we receive $y = \overline{y}(x)$. As we already know, the difference $\overline{y}(x) - y(x)$ is called variation of the function $y(x)$ and is denoted by δy. The variation δy in variational problems play a role analogous to the role of the increment Δx of an independent variable x in problems for study of extreme of function $f(x)$. The variation of function $\delta y = \overline{y}(x) - y(x)$ is a function of the x. This function can be differentiated one or several times, e.g. $(\delta y)' = \overline{y}'(x) - y'(x) = \delta y'$ i.e. it is generated of the variance and it is equal to the variance of the generated, e.g.

$$(\delta y)'' = \overline{y}''(x) - y''(x) = \delta y'',$$

$$\cdots\cdots\cdots\cdots$$

$$(\delta y)^{(k)} = \overline{y}^{(k)}(x) - y^{(k)}(x) = \delta y^{(k)}.$$

If we look at the values of functional (0.1) only of the family of curves $y = y(x,\alpha)$, the functional turns into function of α, $v[y(x,\alpha)] = \varphi(\alpha)$, as in the case that we consider $v[y(x,\alpha)]$ as functional depending on parameter, the value of the parameter α determines the curve of the family $y = y(x,\alpha)$ which on other hand determinesthe value of functional $v[y(x,\alpha)]$.

<u>Theorem 1.</u>

If functional $v(y) = \int_{x_0}^{x_1} F(x, y, y')dx$ has a local extreme in y, the necessary condition for extreme of functional is

$$\int_{x_0}^{x_1} [F_y - \frac{d}{dx} F_{y'}]\delta y\, dx = 0, \qquad (0.2)$$

<u>Proof of theorem 1 .</u>

We analyze the function $\varphi(\alpha)$. It reaches its extreme at $\alpha = 0$, and when $\alpha = 0$ we obtain $y = y(x)$. By assumption the functional reaches its extreme compared with any permissible curve, and in particular, in terms of the nearly families curves $y = y(x,\alpha)$. Necessary condition for extreme of the function $\varphi(\alpha)$ at $\alpha = 0$, as is known, is its a derivative is equal to zero at $\alpha = 0$, i.e. $\varphi'(0) = 0$. Since

$$\varphi(\alpha) = \int_{x_0}^{x_1} F(x, y(x,\alpha), y_x'(x,\alpha))\,dx,$$

$$\varphi'(\alpha) = \int_{x_0}^{x_1} \left[F_y' \frac{\partial}{\partial\alpha} y(x,\alpha) + F_{y'}' \frac{\partial}{\partial\alpha} y'(x,\alpha)) \right] dx,$$

where

$$F_y' = \frac{\partial}{\partial y} F(x, y(x,\alpha), y'(x,\alpha)),$$

$$F_{y'}' = \frac{\partial}{\partial y'} F(x, y(x,\alpha), y'(x,\alpha)),$$

$$\frac{\partial}{\partial\alpha} y(x,\alpha) = \frac{\partial}{\partial\alpha}[y(x) + \alpha\delta y] = \delta y$$

$$\frac{\partial}{\partial\alpha} y'(x,\alpha) = \frac{\partial}{\partial\alpha}[y'(x) + \alpha\delta y'] = \delta y',$$

and we get

$$\varphi'(\alpha) = \int_{x_0}^{x_1} \Big[F_y(x, y(x,\alpha), y'(x,\alpha))\delta y +$$

$$F_{y'}(x, y(x,\alpha), y'(x,\alpha))\delta y' \Big] dx,$$

$$\varphi'(0) = \int_{x_0}^{x_1} \Big[F_y(x, y(x), y'(x))\delta y +$$

$$F_{y'}(x, y(x), y'(x))\delta y' \Big] dx \quad (npu\ \alpha = 0).$$

As we know, $\varphi'(0)$ is called variation of functional and means δv. Necessary condition for extreme of functionalis its variation to be equal to zero $\delta v = 0$. For the functional (0.1) this condition has a type of

$$\int_{x_0}^{x_1} [F_y'\delta y + F_{y'}'\delta y']\,dx = 0 \qquad (0.3)$$

Integrating the equation (0.3) by parts, where $\delta y' = (\delta y)'$, we get

$$\delta v = [F_{y'}'\delta y']\Big|_{x_0}^{x_1} + \int_{x_0}^{x_1} [F_y' - \frac{d}{dx}F_{y'}']\delta y\,dx =$$

$$= \int_{x_0}^{x_1} F_y'\delta y\,dx + F_{y'}'(x_1, y(x_1,\alpha), y'(x_1,\alpha))\delta y(x_1) -$$

$$- F_{y'}'(x_0, y(x_0,\alpha), y'(x_0,\alpha))\delta y(x_0) =$$

$$= \int_{x_0}^{x_1} F_y'\delta y\,dx + F_{y'}'(x_1, y(x_1,\alpha), y'(x_1,\alpha))(\bar{y}(x_1) - y(x_1))$$

$$- F_{y'}'(x_0, y(x_0,\alpha), y'(x_0,\alpha))(\bar{y}(x_0) - y(x_0)) - \int_{x_0}^{x_1} (\delta y)dF_{y'}' =$$

$$= \int_{x_0}^{x_1} F_y'\delta y\,dx + F_{y'}'(x_1, y(x_1,\alpha), y'(x_1,\alpha))(0)$$

$$- F_{y'}'(x_0, y(x_0,\alpha), y'(x_0,\alpha))(0) - \int_{x_0}^{x_1} (\delta y)\frac{d}{dx}F_{y'}'$$

Since, all of the possible (permissible) curves in the given problem pass through fixed limit points, we get

$$\delta v = \int_{x_0}^{x_1} [F_y' - \frac{d}{dx}F_{y'}']\delta y\,dx.$$

To simplified the obtained necessary condition (0.2), we will use the following lemma:

<u>Fundamental lemma of the variational calculus</u>

If for any continuous function $\eta(x)$ it is true that

$$\int_{x_0}^{x_1} \Phi(x)\eta(x)\,dx = 0,$$

where the function $\Phi(x)$ is continuous in the interval $[x_0, x_1]$, then

$$\Phi(x) \equiv 0$$

in this interval.

Proof of the fundamental lemma of variational calculus

We accept that, in the point $x=\bar{x}$, resting in the interval (x_0, x_1) , $\Phi(x)\neq 0$,is a contradiction. Indeed, the continuity of the function $\Phi(x)$, it follows that if $\Phi(\bar{x})\neq 0$ it $\Phi(x)$ keeps characters in vicinity of $\bar{x}$ ($x_0 \le x \le x_1$). We choose function $\eta(x)$ which also retains the mark in that vicinity and is equal to zero outside of this vicinity. We receive

$$\int_{x_0}^{x_1} \Phi(x)\eta(x)\,dx = \int_{\bar{x}_0}^{\bar{x}_1} \Phi(x)\eta(x)\,dx \neq 0,$$

Since product $\Phi(x)\eta(x)$ retains its mark in the interval $x_0 \le x \le x_1$ and is equal to zero in the same interval. And so, we come to a contradiction, therefore $\Phi(x)\equiv 0$.

Note. Adoption of lemma and its proof remain unchanged if the function $\eta(x)$ requires the following restrictions:

$\eta(x_0)=\eta(x_1)=0,$

$\eta(x)$ There is a continuous derived to line n ,

$\left|\eta^{(s)}(x)\right| < \varepsilon, \quad (s=0,1,\ldots,q; q\le n)$.

The function $\eta(x)$ can be selected, e.g. :

$$\eta(x)=\begin{cases} k(x-\bar{x}_0)^{2n}(x-\bar{x}_1)^{2n}, & x\in[\bar{x}_0,\bar{x}_1] \\ 0 & x\in[x_0,x_1]\setminus[\bar{x}_0,\bar{x}_1] \end{cases},$$

where n is a positive number, k is a constant.

Apparently, that the function $\eta(x)$ satisfies the above conditions: it is a continuous, there is a continuous derived to line $2n-1$, in the points x_0 and x_1 is equal to zero and by reducing the factor by k we can do $\left|\eta^{(s)}(x)\right| < \varepsilon$ for the $\forall x\in[x_0, x_1]$. Now we will apply the fundamental lemma of variational calculus to simplify the above necessary condition for extreme (0.2) of functional (0.1).

Consequence1.1.

If the functional $v(y)=\int_{x_0}^{x_1} F(x,y,y')dx$ reaches extreme of the curve $y=y(x)$, and F_y' and are $\frac{d}{dx}F_{y'}'$ continuous, then it $y=y(x)$ is a solution to the differential equation (equation of Euler)

$$F_y - \frac{d}{dx}F_{y'} = 0,$$

Or in an expanded form

$$F_y - F_{xy'} - F_{yy'}y' - F_{y'y'}y'' = 0 .$$

This equation is called equation of Euler (1744 year). Integral curve $y=y(x,C_1,C_2)$ equation of Euler is called extreme. To find a curve, which is reached extreme of functional(0.1) we integrate the equation of Euler and spell out random constants, satisfying the general solution of this equation, of the conditions of borders $y(x_0)=y_0, y(x_1)=y_1$. Only if they are satisfied with these conditions, the extreme of the functionalcan be reached. However, in order to determine whether they are really extreme (maximum or minimum), we must investigate the sufficient conditions for extreme as well. To recall, that boundary problem

$$F_y{}' - \frac{d}{dx}F_{y'}' = 0, \quad y(x_0)=y_0, \quad y(x_1)=y_1,$$

not always has a solution, and if there is a solution, then this may not be unique. It should be taken into account that in many variational problems the existence of solutions is evident, from physical or geometrical sense of the problem, and in the solution of the equations of Euler satisfying the border conditions, only a single extreme may be the solution of the given problem.

Case study

Problem of the smallest surfaces in rotation

We want to determine curve at a given boundary points, by rotation around an axis which on the x-axis forms the smallest area.

It is known that the surface area of rotational body is

$$S[y(x)] = 2\pi \int_{x_0}^{x_1} y\sqrt{1+y'^2}\,dx \cdot$$

The integrand function depends only on y and y', therefore, the integral of the equation of Euler will have a type

$$F - y'F'_{y'} = C_1,$$

Or in this case $y\sqrt{1+y'^2} - \frac{yy'^2}{\sqrt{1+y'^2}} = C_1 \cdot$

After a simplification,

$$\frac{y(1+y'^2) - yy'^2}{\sqrt{1+y'^2}} = C_1$$

$$\Leftrightarrow \frac{y + yy'^2 - yy'^2}{\sqrt{1+y'^2}} = C_1$$

We receive $\frac{y}{\sqrt{1+y'^2}} = C_1 \cdot$

The easiest way to integrate this equation is by the application $y' = sht$, where $y = C_1 cht$, and

$$dx = \frac{dy}{y'} = \frac{C_1 sht\, dt}{sht} = C_1\, dt$$

$$\Rightarrow x = C_1 t + C_2$$

And so, the demandsurface forms a curve of rotation, the equation in parametric form has the type

$$x = C_1 t + C_2$$
$$y = C_1 cht$$

Excluding the parameter t, we have $y = C_1 ch\left(\frac{x - C_2}{C_1}\right)$- family curves, from whose rotation is forms a surface, which is called Catenoid (picture 1). Constants C_1 and C_2 are determined by the conditions atthe passing of the curve in the given boundary points (depending on the position of the points, there may be one, two, or neither one decision).

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